Trade and Domestic Production Networks*

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Abstract

In this paper we study how international trade affects firm efficiency and real wages. Both in our theory and in the Belgian data, firms trade with each other, and external shocks transmit along the firm-to-firm production network. We first document that the exposure to foreign inputs is much larger when network effects are taken into account compared to a standard analysis in which only direct importing or a common intermediate good is considered. We then develop and estimate a tractable model of domestic firm-to-firm trade, external trade, and endogenous network formation. Finally, we study the transmission of a foreign trade shock through the Belgian economy.

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1 Introduction

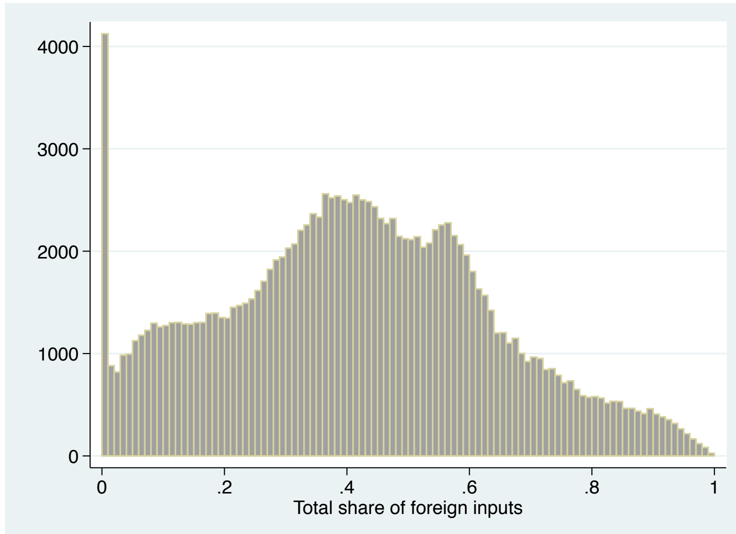
Globalization is under fire from populist groups. It has long been recognized that part of the backlash against globalization is due to a lack of visibility of the beneficiaries of international trade (Friedman, 1978). Only few firms directly export or import foreign goods. Clearly, firms do not need to import themselves to benefit from cheaper foreign inputs; they can simply purchase goods from another firm that is using foreign inputs directly or indirectly. Usually these firm-to-firm trade flows are unobserved to the outsider and researcher, making it difficult to quantify the distribution of cost savings due to importing across firms.

In this paper, we make use of a rich data set of all domestic firm-to-firm transactions in Belgium – informing us about the domestic firm-to-firm production network – as well as their annual accounts, and custom records on firm-level imports and exports. To fix ideas, we define a firm's total share of foreign inputs as the sum of its direct share of foreign inputs and the share of its inputs from other firms, multiplied by those firms' total share of foreign inputs. We display the histogram of the total share of foreign inputs by private sector Belgium firms in Figure 1. It is notable that for the median firm, the *total* foreign input share is 41 percent, while for 85 percent of the Belgium private sector firms the *direct* foreign input share (not shown in figure) is zero. Hence, a large fraction of firms is heavily dependent on foreign inputs, while for only few importing firms this dependence on foreign inputs is visible from their direct foreign input shares.

To analyze the implications of domestic firm-to-firm trade for the transmission of foreign trade shocks, we develop a model of domestic firm-to-firm trade and international trade. In our model, firms can use foreign inputs, domestic inputs produced by other firms, and labor to produce. Firms are finite and monopolistically competitive. We distinguish between the cases of a fixed network structure (e.g., the buyer-supplier-pairs are fixed) and flexible network structure (e.g., firms endogenously choose from which other firm to purchase inputs). Assuming that the production function has the CES property, we show that, given a fixed network structure, the total foreign input share defined above and the elasticity of substitution in the production function are sufficient to calculate the cost reductions due to foreign inputs. Given plausible values of the elasticity of substitution in the production function of around 2 (or 1.5), shutting down international trade would increase the cost of the median Belgium firm by about 70 (or 289) percent.

An increase in the barriers to trade may not only affect the intensive margin of purchases given a network of suppliers, but also the extensive margin of firm-to-firm linkages. In the model with endogenous network structure, firms optimally choose their set of suppliers (which we label the firm's sourcing strategy) and incur a firm-pair-specific fixed cost for adding a supplier. Solving a model of firm-to-firm network formation is arguably difficult. In particular, two challenges stand out: 1) Firms face a large discrete choice problem of which

Figure 1: Histogram of total share of foreign inputs



Notes: Total share of foreign inputs of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct share of foreign inputs, and s_{ji} is j's share among i's inputs. The figure is based on the analysis of 139,605 private sector firms in Belgium in 2012. The bar at the far left of histogram, above a total share of foreign inputs of zero, represents firms that use only labor input and do not have any domestic or foreign input purchases.

suppliers to include in their sourcing strategy; 2) Firms' sourcing strategies are interdependent: a single firm's sourcing choice affects its own costs, which affects the benefits to other firms of sourcing from that firm, and which in turn affects the benefits to the firm of sourcing from those other firms. Using the insights from Jia (2008) and Antràs, Fort and Tintelnot (2016) described further below, we overcome the first challenge. We overcome the second challenge by considering the formation of an acyclic network.[1] We postulate an ordering of firms and restrict the eligible set of suppliers to firms appearing in the order before the purchasing firm. While restrictive, making this assumption allows us also to solve a model of domestic firm-to-firm network formation with directed search in addition to solving a model with a general but fixed domestic firm-to-firm network structure. We are in the process of evaluating the robustness of our findings under a fixed network structure in the case of a flexible network structure.

We also provide non-structural evidence that firms react to trade shocks affecting their own customers (suppliers), while controlling for trade shocks that affect all their potential customers (suppliers) as well as the firm's own exports and imports. We follow the approach proposed by Hummels, Jørgensen, Munch and Xiang (2014) to consider as trade shocks changes to world export supply and world import demand of goods in countries that a firm had a previous trade relationship with. The idea is that firm-country-product trade rela-

[1]See Spiegler (2016) for a recent contribution in economics studying belief formation in a directed acyclic network.

tionships are sticky and affect firms more with previous exposure to trading these products with those countries. Furthermore, changes in world export supply and demand are plausibly exogenous. Specifically – and given the other controls discussed above – we find that if a firm's customer (supplier) receives a positive export (import) shock, the firm tends to increase its own scale, i.e., purchases more domestic inputs or has larger domestic sales. This non-structural evidence corroborates our structural evidence that firm-to-firm linkages are important for the transmission of foreign trade shocks.

Our paper is related to three distinct literatures. First, we contribute to the growing literature on the economy-wide effects of foreign sourcing.[2] This literature so far did not have access to domestic firm-to-firm trade flows, and therefore had no precise information on the linkages between domestic firms. Blaum, Lelarge and Peters (2016) provide a sufficient statistics formula for the firm-level cost reductions from importing, in a model in which firms can import directly and purchase a common intermediate good. We extend their sufficient statistic result to the case of a more general firm-to-firm domestic production network and show that the total foreign input share defined above is critical for measuring the cost reductions due to foreign inputs. We also go beyond the fixed network structure assumed in their paper, by solving the endogenous network formation problem. We here borrow from the insights in Jia (2008) and Antràs et al. (2016) to use lattice theory to solve a large combinatorial discrete choice problem. While Antràs et al. (2016) consider a distinction in final good and intermediate good sectors, we consider a more general input-output structure between firms and the formation of not only firms' international sourcing strategies but also firms' domestic sourcing strategies.[3]

Second, we relate to the literature on domestic production networks.[4] Bernard, Moxnes and Saito (2016a) adapt the Antràs et al. (2016) model to search for domestic suppliers in different locations, where in each location there is a continuum of intermediate-good-producing firms. They find significant improvements in firm performance from a reduction in internal search costs in Japan. Furusawa, Inui, Ito and Tang (2017) develop a variant of the global sourcing model by Antràs et al. (2016) and use Japanese buyer-seller link data to test the model's predictions. Oberfield (2017) develops a theory in which the network structure of

[2]Earlier important contributions to the literature on foreign sourcing include Antràs and Helpman (2004), Antràs and Helpman (2008), Rodríguez-Clare (2010), Garetto (2013), Halpern, Koren and Szeidl (2015), Gopinath and Neiman (2014), Amiti and Konings (2007), Goldberg, Khandelwal, Pavcnik and Topalova (2010), De Loecker, Goldberg, Khandelwal and Pavcnik (2016).

[3]Our work is also related to the analysis in Caliendo and Parro (2015) and Ossa (2015). They find the gains from trade to be larger when taking sectoral input-output linkages into account. The firm-level linkages in our paper are of course much more granular and heterogenous than input-output tables. We also allow the linkages and shares of spending on other firms to change when changing trade openness, while the sectoral input-output linkages are commonly assumed to be fixed.

[4]A growing body of work studies how firms meet international trading partners. See for example Chaney (2014), Chaney (2016), Morales, Sheu and Zahler (2015), Eaton, Kinkins, Tybout and Xu (2016).

production forms endogenously among firms that each purchase a single input. Lim (2015) develops a dynamic model of network formation in which each firm has a continuum of domestic suppliers. With a continuum of suppliers and firms, all the firm-to-firm shares are neglibibly small and a link between two particular firms has no effect on aggregate outcomes. In contrast to these papers, we develop a model of formation of links with a finite set of suppliers and incorporate both firm exporting and importing.

Finally, we relate to the literature that analyzes the macro implication of micro shocks. Gabaix (2011) provides conditions under which granular shocks can affect aggregate fluctuations. Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012) study the transmission of shocks along sectoral input-output networks. Magerman, De Bruyne, Dhyne and Van Hove (2016) test both channels with the Belgium domestic firm-to-firm data. Barrot and Sauvagnat (2016), Boehm, Flaaen and Pandalai-Nayar (2015), and Carvalho, Nirei, Saito and Tahbaz-Salehi (2014) use natural disasters to study the propagation of shocks in production networks. Carvalho and Voigtländer (2015) analyze the adoption of inputs by innovators and the evolution of the domestic production network. Hulten (1978) provides conditions under which the underlying network structure is irrelevant for quantifying the propagation of shocks – up to a first-order approxmiation – as long as firms' initial size and the magnitudes of the idiosyncratic shocks are observed. In recent work, Baqaee and Farhi (2017) illustrate that the second-order effects of shock propagation arising from networks can be large. [5] In this paper, we extend the analysis of shock propagation to a foreign trade shock. We find empirically that the effects of a foreign trade shock are larger under the actual observed firm-to-firm trade structure than in an analysis with the same intermediate input shares but a common intermediate good.

The rest of the paper is organized as follows. Section 2 outlines the model. Section 3 describes the data and provides descriptive results on the network structure and on the propagation of shocks through the network. Section 4 estimates the parameters of the model, and Section 5 conducts counterfactual exercises with endogenous network structure. Section 6 concludes.

2 A model of trade and domestic production networks

We develop a model of a small open economy called Belgium. We study how firms form production networks and choose whether to import and/or export. In our model, network formation is a combination of exogenous shocks (i.e., which potential suppliers does a firm meet?) and endogenous choices (i.e., among the set of potential suppliers, which suppliers

[5] Other recent contributions to determining the effects of networks include Baqaee (2014), Carvalho and Grassi (2017), as well as in the context of financial frictions, Bigio and La'o (2016) and Liu (2016).

does a firm choose to buy from, and does a firm export or import?).

Before describing the model, we briefly discuss the notation: Since there exist many bilateral directed flows in our model, we will often have two subscripts. In the case of two subscripts, we use the the first subscript to denote the origin of the good and the second subscript to denote the destination.

We begin by describing consumer preferences and demand.

2.1 Preferences and Demand

Each consumer provides one unit of labor and there is no dis-utility from working. Consumers are assumed to have identical homothetic CES preferences over consumption goods:

$$U = \left(\sum_{k \in \Omega_D} (\beta_{kD} q_{kD})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where Ω denotes the set of available products in the small open economy, k denotes a product, and D denotes domestic final demand. Since all consumers have the same homothetic preferences for consumption, we can write the aggregate final consumer demand (in quantities) for product k, given price p_{kD}, as:

$$q_{kD} = \beta_{kD}^{\sigma-1} \frac{p_{kD}^{-\sigma}}{P_D^{1-\sigma}} E_D, \tag{2}$$

where E_D denotes the aggregate expenditure in Belgium and P_D denotes the domestic consumer price index:

$$P_D = \left(\sum_{j \in \Omega} \beta_{jD}^{\sigma-1} p_{jD}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

We assume that final goods are substitutes and therefore $\sigma > 1$.

Demand from abroad for product k is given by:

$$q_{kF} = \beta_{kF}^{\sigma-1} \frac{p_{kF}^{-\sigma}}{P_F^{1-\sigma}} E_F, \tag{4}$$

where p_{kF} is the price of product k abroad.

Next, we proceed to describe the production structure and the problem of the firm.

2.2 Production and market structure

Firms produce single products. We will use i, j, k to index firms / products. The products are differentiated across firms. Firms sell the same product to final consumers and to other firms as an intermediate input, though not all firms sell to other firms, and not each pair of firms has a buyer-seller relationship. Note that we allow Belgian firms to directly sell to foreign consumers, while all foreign products reach Belgian consumers indirectly through the importing of inputs by Belgian firms.[6]

We treat every firm as infinitisimal when selling to final consumers. Hence, when selling to domestic or foreign final consumers, we assume the market structure is monopolistic competition. When selling to other firms, the assumption of infinitisemal size is no longer reasonable, however, since most firms just have a few selected suppliers. We assume that in the Nash bargaining between buyer and supplier, the buyer has the full bargaining power. Given the assumptions on technology described below, this will imply that the supplier sells at marginal cost to the buyer firm. Firms make their profits from sales to final consumers. The arguably strong assumption of the bargaining power in firm-to-firm trade being on the buyer's side will be critical for modeling the network formation game in a tractable manner.[7]

We assume it is costly for the firm to start purchasing from a supplier, requiring the payment of a buyer-seller-pair-specific fixed cost. We first describe the firm's problem given the selection of a set of suppliers Z_j for firm j, and we then discuss the endogenous formation of the set of suppliers and buyers further below.

2.2.1 Production outcomes given a domestic production network

For the first part, we will take as given the set of firms Z_j from which each firm j is eligible to purchase inputs.

Firms use a CES input bundle of workers and domestic and foreign inputs with elasticity of substitution $\rho > 1$ in the production function. For what follows, we assume that $\sigma > \rho$, hence final consumers are more price elastic than firms purchasing intermediate inputs.

Given the CES production function, we can write the cost function of firm j as:

$$c_j(Z_j) = \frac{1}{\phi_j} \left(\sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{1/(1-\rho)}. \quad (5)$$

Following Antràs et al. (2016), we will call $\Theta_j(Z_j) = \sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}$ the *sourcing capability* of firm j, and Z_j the *sourcing strategy* of firm j. The sourcing strategy may include both domestic and foreign sourcing. The price of labor is denoted by w_ℓ. The

[6]The assumption that foreign goods reach Belgian consumers only through Belgian firms is reasonable because in the data nearly all imports are carried out by firms. We make the assumption that Belgian firms can reach foreign consumers directly to avoid modeling foreign firms in detail.

[7]Even in the industrial organization literature, corner solutions in the bargaining game are sometimes assumed to obtain tractable solutions for network formation problems. For example, when studying the determinants of the hospital networks offered by health plans, Ho (2009) assumes that hospitals make take-it-or-leave-it offers to all health plans in the market.

share of variable costs by firm j that is spent on intermediate inputs produced by firm $k \in Z_j$ is:

$$s_{kj} = \frac{p_{kj} q_{kj}}{c_j q_j} = \frac{\alpha_{kj}^{\rho-1} p_{kj}^{1-\rho}}{\Theta_j(Z_j)}. \tag{6}$$

Analogously, the share of variable costs by firm j that is spent on labor is:

$$s_{\ell j} = \frac{w_\ell \ell_j}{c_j q_j} = \frac{\alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}}{\Theta_j(Z_j)}, \tag{7}$$

while the *direct* share of foreign inputs by firm j (assuming $F \in Z_j$) is:

$$s_{Fj} = \frac{p_{Fj} q_{Fj}}{c_j q_j} = \frac{\alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho}}{\Theta_j(Z_j)}. \tag{8}$$

Before deriving an expression for the total sales of a firm, we next discuss the pricing problem of the firm. Due to CES preferences and monopolistic competition, firms charge a constant mark-up over marginal costs, $\mu = \frac{\sigma}{\sigma-1}$, when selling to final consumers at home or abroad. When selling to other firms, firms engage in Nash bargaining with the full bargaining power on the side of the buying firm. The buyer will make the seller just indifferent between selling to the firm or not, and therefore firms sell at marginal costs to other firms.

In order to sell abroad firms incur iceberg transport costs, τ. For this subsection we take export participation, I_{jF}, as given ($I_{jF} = 1$ for all exporting firms and $I_{jF} = 0$ otherwise) and endogenize it in the following subsection. Firms' total sales consist of the sum of domestic sales to final consumers, foreign sales to final consumers, and domestic sales to other firms. Let firm j's total sales be denoted by:

$$\begin{aligned} x_j =& \beta_{jD}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \frac{E}{P^{1-\sigma}} + I_{jF} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} \\ &+ \sum_k I(j \in Z_k) \mu^{1-\rho} \phi_j^{\rho-1} \alpha_{kj}^{\rho-1} \Theta(j) \frac{x_k/\mu_k}{\Theta_k(Z_k)}, \end{aligned} \tag{9}$$

where μ_k denotes the average mark-up of firm k (recall that the firm charges a constant mark-up to final consumers and a zero mark-up to other firms, hence μ_k depends on the distribution of firm k's sales).

We next calculate the exposure of firms to foreign inputs. It is important to note, that the direct and total share of foreign inputs (including foreign inputs obtained indirectly via domestic suppliers) can be substantially different. Let s_{Fj} denote the *total* share of inputs by firm j that comes from foreign, F:

$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j} s_{ij} s_{Fi}^{Total}. \tag{10}$$

The definition of total share of foreign inputs is intuitive for a model with single product firms in which each firm uses the same fraction of foreign inputs in the production sold to every buyer. It turns out that it can be directly linked to the cost reductions from foreign input – at least in the short-run, in the absence of any changes in firm linkages, and holding other factor prices fixed – as we summarize in the following proposition.

Proposition 1 (Cost increases from banning foreign inputs) *Assume $\rho > 1$. Ignoring linkages and indirect effects (i.e., pretending there is no pass-through of cost changes from domestic suppliers) and leaving domestic nominal wages, w_ℓ, unchanged, the direct cost increase from banning foreign inputs is:*

$$\hat{c}_j \mid_{direct}^{p_{F\cdot} \to \infty} = (1 - s_{Fj})^{1/(1-\rho)}. \tag{11}$$

Given fixed linkages between firms, and leaving domestic nominal wages, w_ℓ, unchanged, the total cost increase from banning foreign inputs is:

$$\hat{c}_j \mid_{total}^{p_{F\cdot} \to \infty} = \left(1 - s_{Fj}^{Total}\right)^{1/(1-\rho)}. \tag{12}$$

The direct cost increase from banning foreign goods result in the proposition is standard and obtained in earlier work by Arkolakis, Costinot and Rodríguez-Clare (2012) and Blaum et al. (2016). To our knowledge, the total cost reduction result is novel and reflects that each firm in general has a different total exposure to foreign inputs even if the total expenditure on intermediates is the same.[8] Clearly, the total and direct cost reduction effect will be equal if there are no domestic intermediate purchases. The result in Proposition 1 is based on the assumption of fixed linkages between firms. We also keep nominal wages unchanged. Later in the paper, we will conduct counterfactual analysis with a changing network structure.

Given that firms make their profits only on sales to final consumers, we can write the *variable* profits of firm j given a sourcing strategy, Z_j, and export participation, I_{jF}, as

[8] In earlier work, Blaum et al. (2016) obtain a total cost reduction result from foreign inputs in which each firm buys the same CES bundle of intermediate inputs. Their production function is Cobb-Douglas in intermediate inputs and own labor input. In Appendix A.2, for the case of roundabout production without firm-to-firm network linkages, we derive a cost reduction result similar to Blaum et al. (2016) that is applicable to a production function which is CES between labor input and intermediates.

$$\pi_j^{var}(Z_j, I_{jF}) = \frac{1}{\mu}\beta_{jD}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}} + I_{jF}\frac{1}{\mu}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}}. \quad (13)$$

The variable profits of firm j given a sourcing strategy, Z_j, and export participation, I_{jF}, are relevant for the firms' endogenous choices of sourcing strategies and export participation.

2.2.2 Determination of firm sourcing strategy, import and export participation

We assume that only buyers search to form linkages with other domestic firms. Forming linkages to suppliers is costly, and firm j incurs a random, firm-pair-specific, fixed cost f_{kj} to add supplier k. The realization of fixed costs is known to the firm at the time at which it selects suppliers. Firms in our model make profits due to positive mark-ups in sales to final consumers. Since by assumption in any firm-to-firm trade the buyer has the full bargaining power, firms do not make profits from sales to other firms. Hence, variable profits are proportional to firm-level sales to final consumers.

We can write the profits of firm j , given a sourcing strategy, Z_j, and a set of fixed unit labor requirements, $\{f_{kj}\}$, to add suppliers and export fixed costs, $f_{jF}w_\ell$, as

$$\pi_j(Z_j, I_{jF}) = \pi_j^{var}(Z_j, I_{jF}) - \sum_{k\in Z_j} f_{kj}w_\ell - I_{jF}f_{jF}w_\ell. \quad (14)$$

We assume that firm j exogenously meets a set of potential suppliers, $\mathbf{Z}_j$. Firm j then endogenously decides on the set of suppliers and on export participation.

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j \ , \ I_{jF} \in \{0, 1\} \quad (15)$$

Analogously to the problem studied by Antràs et al. (2016), under the assumption that demand is more elastic than the elasticity of substitution between inputs in the production function, $\sigma > \rho$, the sourcing decision will be complementary across suppliers as well as complementary to the exporting decisions. In other words, the marginal benefit of adding a supplier is increasing in the set of other suppliers that you are adding and is increasing in the binary export decision. We can therefore apply the same tools as in Antràs et al. (2016), namely the adaption of the Jia (2008) algorithm to the problem of firm sourcing, to solve the problem described in (15). We will discuss the determination of the set of potential suppliers, $\mathbf{Z}_j$, further below.

2.3 Aggregation and equilibrium

We next describe the aggregation of our model to national or regional-level flows, discuss how firm profits are redistributed to consumers and define the equilibrium of our model.

We assume that the set of Belgian firms is fixed and that firm profits are distributed to workers in Belgium. We consider Belgium as a small-open economy and assume that there are no foreign asset holdings and trade is balanced. Hence aggregate expenditure in Belgium is given by

$$E = w_\ell L + \sum_k \pi_k. \tag{16}$$

Balanced trade implies that aggregate exports are equal to aggregate imports:

$$\sum_j I_{jF} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} = \sum_j \frac{\mu_j - 1}{\mu_j} s_{Fj} x_j. \tag{17}$$

Labor market clearing implies that labor income is equal to firms' labor costs:

$$w_\ell L = \sum_j \frac{\mu_j - 1}{\mu_j} s_{\ell j} x_j + w_\ell \sum_j \left(\sum_{k \in Z_j} f_{kj} + I_{jF} f_{jF} \right). \tag{18}$$

We next consider two different cases of the model: Equilibrium with (1) an exogenous and (2) an endogenous network structure.

2.3.1 Equilibrium with exogenous network structure

We first consider the case with an exogenous network structure. Under an exogenous networks structure we abstract away from fixed costs of including suppliers in your sourcing strategy or from the fixed costs of exporting.[9]

Definition 2 (Equilibrium given a fixed network structure) *Given foreign expenditure E_F, foreign price index P_F, and a set of prices by foreign suppliers, $\{p_{kj}\}_{k \in foreign}$, an equilibrium for the model with a fixed network structure and fixed export participation is a set of wages (w_ℓ), price index for the consumer (P), and aggregate expenditure (E) such that equations* (3), (5), (6), (7), (9), (16), (17), *and* (18) *hold.*

An equilibrium given a fixed network structure is relatively easy to calculate since firms will take wages and price index as given, which then directly imply expenditure E (since total profits are proportional to domestic and foreign consumer expenditures on Belgian

[9]We abstract away from fixed costs in this case, since we cannot generally guarantee that the fixed unit labor requirements under an exogenous network structure are less than the size of the labor force.

firms' products), and all the relevant equations are differentiable in these arguments. In fact, given wages, we can show that there is a unique solution for the CES price index, as we will summarize in the following lemma:

Lemma 3 (Uniqueness of P given wages and fixed network structure) *Define a $K \times K$ matrix A where where the (i, j) element is $\phi_j^{\rho-1}\alpha_{ij}^{\rho-1}$ and K denotes the number of Belgium firms. Assume the matrix $\left(I - A'\right)$ is invertible, where I is the identity matrix. Then for fixed wages, there exists a unique solution for the price index defined in* (3).

Lemma (3) is useful because it implies that the equilibrium will be unique for a given set of wages. We do not have a general uniqueness proof given flexible wages. We next proceed to discuss the change in the aggregate price index arising from banning imports.

Proposition 4 (Change in aggregate price index from banning imports) *Given a fixed number of firms and fixed network structure, the price index change from banning imports is summarized as follows. If the price of intermediate goods is assumed to be unchanged, the price index change can be expressed as*

$$\hat{P}\mid_{direct}^{p_{F\cdot}\to\infty} = \left(\sum_i s_{iH}\left(\hat{c}_i\mid_{direct}^{p_{F\cdot}\to\infty}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{19}$$

If the full effect of firm-to-firm linkages with flexible prices for intermediate goods is taken into account, the expression becomes

$$\hat{P}\mid_{total}^{p_{F\cdot}\to\infty} = \left(\sum_i s_{iH}\left(\hat{c}_i\mid_{total}^{p_{F\cdot}\to\infty}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{20}$$

Finally, if one assumes roundabout production as in Blaum et al. (2016), then the expression becomes

$$\hat{P}\mid_{roundabout}^{p_{F\cdot}\to\infty} = \left(\sum_i s_{iH}\left(\hat{c}_i\mid_{roundabout}^{p_{F\cdot}\to\infty}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{21}$$

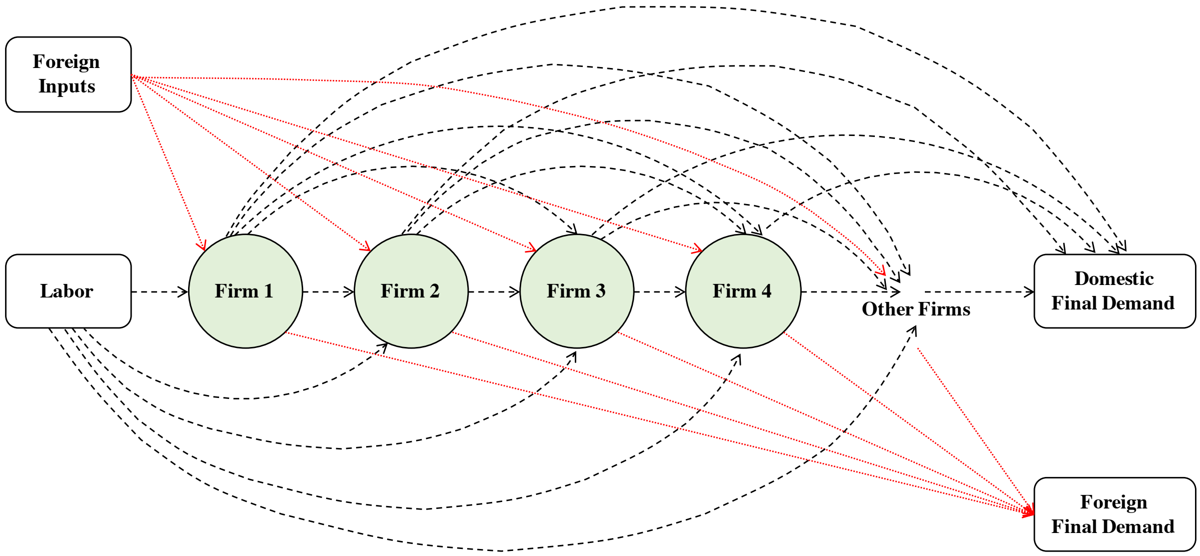
See Appendix section A.2 for the derivation of $\hat{c}_i\mid_{roundabout}^{p_{F\cdot}\to\infty}$.

We continue with discussing the formation of the endogenous network structure.

2.3.2 Equilibrium with endogenous network structure

The formation of links between firms are endogenous choices themselves. Importantly, the choice of links domestic links between firms may be a function of the foreign trade opportunities as well.

Figure 2: Endogenous network formation – eligible connections



When solving the problem for every firm described in (15), a key issue that arises is that each firm needs to guess a set of costs for its potential suppliers, where the costs themselves are equilibrium objects and depend on everyone's sourcing decision. Not only is it an extremely challenging task, computationally, to find a fixed point in the set of costs for all firms so that these costs are consistent with everyone's optimal sourcing decision, but also the uniqueness of such a fixed point is rather unlikely. For example, if firms guess that the suppliers have high unit costs, the firms may not source from many suppliers, which then indeed could lead to high unit costs. Conversely, guessing that firms have very low unit costs could result in the formation of many linkages and lead to lower unit costs overall. To get around these problems, we propose the following mechanism according to which firms choose suppliers. The mechanism relies on the assumption of sequentiality and that some firms are more constrained in the set of firms they can source from than others. The firm's order in the sequence of supplier choices and its set of eligible suppliers are becoming attributes of the firm and therefore primitives of the model.

We put firms into a sequence $S = \{1, 2, 3, ..., N\}$ that determines the order in which they can choose their suppliers and also affects the set of eligible suppliers. The procedure works as follows (see also the illustration in Figure 2): Firm 1 moves first and can only hire labor input. Firm 2 moves second and can hire both labor input as well as source the input produced by firm 1. Firm 3 can choose between labor and both firm 1 and firm 2 inputs. And so on. Firms much later in the sequence will have a very large set of eligible suppliers. While the adaption of Jia's algorithm to the problem of sourcing can help us solve very large interdependent discrete choice problems, we are still limited in the number of possible sourcing strategies we can feasibly evaluate. We therefore restrain the set of potential suppliers for firm j, $\mathbf{Z}_j$, to be a random subset from the set of eligible suppliers. The suppliers for firm j are then optimally chosen as the solution to the problem in (15). In practice, we choose the cardinality of $\mathbf{Z}_j$ to be 200, so the firm still chooses among 2^{200} or 10^{60} possible supplier sets. By following Antràs et al. (2016)'s adaption of the Jia algorithm, we are able to tractably solve that large discrete choice problem.[10]

The assumption made earlier that for any firm-to-firm trade the buyer has the full bargaining power is important for the tractability of the network formation problem. Even when putting firms on a sequence, in principle firms would need to evaluate the decisions of other firms coming later in the sequence if they could make profits on these transactions. However, given that here they cannot make profits off the firms coming later in the sequence, they only need to know the choices of firms earlier in the sequence, leading to a tractable network formation problem.

The sequential nature of the problem, together with the fact that there is a unique solution to the problem of (15) (after imposing a tie-breaking rule that in the case of indifference a supplier is included), the network formation given a set of wages and a guess for the price index will be unique. We can then alter wages, price index, and expenditure to achieve labor market clearing, trade balance, and a fixed point for the price index and expenditure.

Obtaining general results in the case of an endogenous network structure is difficult. We will later structurally estimate the endogenous network formation model and provide numerical results for counterfactual analysis.

3 Data sources and descriptive evidence

3.1 Data description

Our data comes from five main sources. The first source is the Business-to-Business (B2B) transactions database (see also See Dhyne, Magerman and Rubinova (2015)), which records the universe of Belgian enterprise-to-enterprise annual transactions from 2002 to 2014. All Belgian enterprises are assigned VAT-ids, and are required to file their annual sales to other VAT-ids, as long as the annual sales exceed 250 Euro. Our second source is another data set of VAT declarations. In addition to filing B2B sales, all Belgian enterprises that are VAT liable have to report their total sales and their total purchases in their VAT declarations. The VAT declaration contains the total sales value, the VAT amount charged on those sales (both to enterprises and to final consumers), the total amounts paid on inputs sourced and the VAT paid on those inputs. Each enterprise has to file this declaration either on a monthly or a quarterly basis depending on some thresholds, and our data set contains the annualized

[10] We describe the computational algorithm to solve for the equilibrium in section B in the Appendix.

Table 1: Number of VAT-ids in multiple VAT-id firms

	Mean	10%	25%	50%	75%	90%	max
Num. vat-id	3	2	2	2	3	4	372

values. Our third source is a data set of the annual accounts filed by Belgian enterprises. These data inform us about a VAT's total output, labor cost, ownership of other VATs, zip code, and 4-digit NACE classification of the industry. Our fourth source is Belgian customs data containing VAT-year-country-product-level international trade transactions. See Appendix C.3 for the reporting thresholds for the international trade data set.[11] The fifth data source is the Balance of Payments survey, which allows to identify international financial participations (inward and outward FDI).

The above data sets are all recorded at the VAT-id level. While other papers that use Belgian firm level data (such as Amiti, Itskhoki and Konings (2014), Magerman et al. (2016) and Bernard, Blanchard, Van Beveren and Vandenbussche (2016b)) analyze directly the VAT-id-level information, there are no explicit rules that one firm can only have one VAT-id. In fact, many large firms tend to allocate parts of their operations into multiple VAT-ids. While within-firm B2B transactions are interesting by themselves, in this paper we are primarily interested in inter-firm trade. Therefore we aggregate VAT-ids up to the firm level, using ownership filings in the Annual Accounts and the Foreign Direct Investment data. In the ownership filings, each enterprise reports a list of all other enterprises of which it has a positive share, and the value of the share. In the FDI data, we observe for each Belgian enterprise whose shares are owned to at least 10% by a foreign firm, the names of its foreign parent firm and the country the parent firm is located in, along with the ownership share. We group all VAT-ids into firms if they are linked with more than or equal to 50% of ownership, or if they share the same foreign parent firm that holds more than or equal to 50% of ownership. Details of the aggregation are outlined in the appendix. After this aggregation procedure, around 896,000 VAT-ids collapsed to 860,000 firms in 2012. Of those firms, 842,000 firms consisted of single VAT-ids. The 18,000 firms with multiple VAT-ids account for around 60% of the total output in the data set. The number of VAT-ids for the multiple VAT-id firms are shown in table 1.

After we have converted our data sets from the VAT-id level to the firm level, we restrict our primary sample of analysis according to the following criteria: we include Belgian firms with positive labor cost and in industries other than government and finance, which report

[11]We observe the primary sector of each VAT-ids in terms of the statistical classification of economic activities in the European Community (NACE) Rev.2, at the level of 4 digits. However, the international trade data set categorizes products at the Combined Nomenclature (CN) 8 digits. Therefore we convert the CN 8 digit products into NACE Rev.2 4 digit, and conduct all analysis with the NACE Rev.2 classification. See Appendix C.4 for details.

positive employment, tangible assets of more than 100 Euro, and positive total assets for at least one year throughout our sample period. These criteria are similar to the ones used by De Loecker, Fuss and Van Biesebroeck (2014). Applying these criteria reduces the number of firms in 2012 to around 140,000. Table 2 illustrates that these selected firms provide a good coverage of our sample in terms of value added, gross output, exports and imports. The total sales in our sample appears to be larger than what is reported in the national statistics. This difference arises from the fact that the output data in the national statistics use value added numbers for trade intermediaries. See Appendix C.2 for the same statistics for all Belgian firms.

Table 2: Coverage of selected sample

Year	GDP (Excl.	Output Gov. & Fin.)	Imports	Exports	Selected sample				
					Count	V.A.	Sales	Imports	Exports
2002	149	411	210	229	122,460	123	516	179	189
2007	192	546	300	314	136,370	157	713	280	269
2012	212	626	342	347	139,605	170	786	296	295

Notes: All numbers except for Count are denominated in billion Euro in current prices. Belgian GDP and output are for all sectors excluding public and financial sector. See Appendix C.2 for the same statistics for the total economy. Data for Belgian GDP, output, imports and exports are from Eurostat.

3.2 Trade shocks and the production chain

In order to illustrate how foreign trade shocks transmit through the production chain, we analyze how firms are affected by trade shocks that hit their customers and suppliers.

In our main specification, we regress changes in firm level outcomes such as domestic sales, labor costs, and domestic input purchases, on its customers' and its suppliers' trade shocks. In addition, we control for the trade shocks on the firm's potential suppliers and customers as well as trade shocks that affect the firm directly (through it's direct import demand or supply). We investigate whether firms' production network plays a role in transmitting shocks, by focusing on whether the shocks that the firm's actual suppliers and customers received have any explanatory power on firm's variables, controlling for the shocks that all the other firms that the firm could have connected to received. If firm-to-firm relationships did not matter, shocks to a firm's supplier / customer should be immaterial to the firm after controlling for the shock to its potential suppliers and potential customers.

We consider a reduced form regression where on the RHS we have exogenous changes in export supply / import demand of the firm's customers and suppliers, on the firm's potential customers and suppliers, and on the firm itself:

$$\begin{aligned}\Delta \log Y_{it} =& \beta + \beta_X^C \Delta \log X_{it}^{IV,C} + \beta_M^S \Delta \log M_{it}^{IV,S} \\ &+ \beta_X^{TC} \Delta \log X_{it}^{IV,TC} + \beta_M^{TS} \Delta \log M_{it}^{IV,TS} \\ &+ \beta_X \Delta \log X_{it}^{IV} + \beta_M \Delta \log M_{it}^{IV} + \varphi_t + \epsilon_{it}. \end{aligned} \tag{22}$$

$\Delta \log X_{it}^{IV,C}$ and $\Delta \log M_{it}^{IV,S}$ each represent the exogenous changes in exports of firm i's customers and imports of firm i's suppliers. $\Delta \log X_{it}^{IV,TC}$ and $\Delta \log M_{it}^{IV,TS}$ each represent the exogenous changes in exports of firm i's potential customers and imports of firm i's potential suppliers. Finally, $\Delta \log X_{it}^{IV}$ and $\Delta \log M_{it}^{IV}$ each represent the exogenous changes on firm i's exports and imports.

To construct these instruments that caputure exogenous trade shocks, we follow Hummels et al. (2014). We construct instrumental variables for changes in firms' exports and imports with changes in world import demand and world export supply, which are correlated with exports and imports but uncorrelated with firms' productivity. Consider firm i's yearly change in exports, $\Delta \log X_{it}$, from $t-1$ to t. The instrument for this variable becomes $\Delta \log X_{it}^{IV}$, which is the log change in world import demand, fixing the firm's exposure to each country-product fixed at the initial period $t-1$:

$$\Delta \log X_{it}^{IV} = \underbrace{\log \sum_{k,c} s_{k,c,t-1}^{i,X} \text{WID}_{k,c,t}}_{X_{it,t-1}^{IV}} - \underbrace{\log \sum_{k,c} s_{k,c,t-1}^{i,X} \text{WID}_{k,c,t-1}}_{X_{it-1,t-1}^{IV}} .$$

$s_{k,c,t-1}^{i,X}$ is the share of exports of firm i at year $t-1$ that falls on product k sold to country c, and $\text{WID}_{k,c,t}$ is the world import demand (excluding imports from Belgium) of country c for product k.[12] Analogously, we construct the instrument for firm i's yearly change in imports, $\Delta \log M_{it}$, with the log change in world export supply:

$$\Delta \log M_{it}^{IV} = \underbrace{\log \sum_{k,c} s_{k,c,t-1}^{i,M} \text{WES}_{k,c,t}}_{M_{it,t-1}^{IV}} - \underbrace{\log \sum_{k,c} s_{k,c,t-1}^{i,M} \text{WES}_{k,c,t-1}}_{M_{it-1,t-1}^{IV}},$$

where $s_{k,c,t-1}^{i,M}$ is the share of imports of firm i at the initial year $t-1$ that falls on product k from country c, and $\text{WES}_{k,c,t}$ is the world export supply (excluding sales to Belgium) of country c for product k.

Notice that the first terms of the previous two equations use the contemporaneous WID / WES but using firm's shares at the previous period, and the second terms use both WID

[12] We use NACE 4 digit level to classify products k.

/ WES and shares at the previous period. By fixing firm's shares for each country-product and allowing variations to come from only the changes in WID / WES, these terms capture exogenous changes in export supply and import demand for each firm. We denote by $X^{IV}_{it,t-1}$ and $M^{IV}_{it,t-1}$ the instrumental variables for exports and imports, using *previous* period shares. Analogously we denote $X^{IV}_{it-1,t-1}$ and $M^{IV}_{it-1,t-1}$ as the instrumental variables for previous period's exports and imports, using previous period shares. Using previous period shares is important for exogeneity of the instrument, since the change in the shares itself is endogenenous.

Using these variables, we next construct instrumental variables that capture the trade shocks that firms' customers and suppliers received. For firm i's customers, we construct their weighted average export demand shocks, $\Delta \log X^{IV,C}_{it}$, using i's initial year output share to each customer firms as corresponding weights:

$$\Delta \log X^{IV,C}_{it} = \log \sum_j \frac{\text{Sales}_{ijt-1}}{\text{TotalSales}_{it-1}} X^{IV}_{jt,t-1} - \log \sum_j \frac{\text{Sales}_{ijt-1}}{\text{TotalSales}_{it-1}} X^{IV}_{jt-1,t-1}.$$

Analogously for the average import supply shocks that i's suppliers received, $\Delta \log M^{IV,S}_{it}$, we have

$$\Delta \log M^{IV,S}_{it} = \log \sum_k \frac{\text{Sales}_{kit-1}}{\text{TotalInputs}_{it-1}} M^{IV}_{kt,t-1} - \log \sum_k \frac{\text{Sales}_{kit-1}}{\text{TotalInputs}_{it-1}} M^{IV}_{kt-1,t-1}.$$

Finally, we construct the instrumental variables that capture the trade shocks that firms' potential customers and suppliers received. Let us define the instrumental variable for export demand shocks that firm i's potential customers received as $\Delta \log X^{IV,TC}_{it}$, and the instrumental variable for import supply shocks that firm i's potential suppliers received as $\Delta \log M^{IV,TS}_{it}$. These potential customers and suppliers include firms that i is actually connected to, and also firms that firm i could have connected to. To determine how much weight firm i assigns to all other firms as potential suppliers and customers, we use the following information. For the weights that firm i assigns to each sector, we use firm i's output and input shares for the sectors. Let sectors, at the NACE 4 digit level, be denoted with u. The output share to sector u for firm i, $\tilde{s}_{iut}$, is defined as the share of i's output that were sold to firms producing sector u goods, and the input share from sector u for firm i, s_{iut}, is defined as the share of inputs of i that came from firms producing sector u goods.

$$\tilde{s}_{iut} = \sum_{j \in W_{it}^u} \frac{\text{Sales}_{ijt}}{\text{TotalSales}_{it}}$$
$$s_{uit} = \sum_{j \in Z_{it}^u} \frac{\text{Sales}_{jit}}{\text{TotalInputs}_{it}}.$$

The term W_{it}^u denotes the set of customers of i producing sector u goods at time t, and Z_{it}^u denotes the set of suppliers of i producing sector u goods at time t. We fix all weights at the previous year $t-1$.

$$\Delta \log X_{it}^{IV,TC} = \log \sum_u \tilde{s}_{iut-1} X_{ut,t-1}^{IV,-i} - \log \sum_u \tilde{s}_{iut-1} X_{ut-1,t-1}^{IV-i}$$
$$\Delta \log M_{it}^{IV,TS} = \log \sum_u s_{uit} M_{ut,t-1}^{IV,-i} - \log \sum_u s_{uit-1} M_{ut-1,t-1}^{IV-i}.$$

$X_{ut,t-1}^{IV,-i}$ and $M_{ut,t-1}^{IV,-i}$ are the instrumental variables for average exports and imports at the sector level, using previous period shares and contemporaneous levels of WID /WES. $X_{ut-1,t-1}^{IV,-i}$ and $M_{ut-1,t-1}^{IV,-i}$ are using both previous period shares and WID /WES.[13]

The coefficients of our main interest in equation (22) are β_X^C and β_M^S: the ones on the shocks on the exports of the firm's customers and imports of firm's suppliers. We include year fixed effects and also truncate outliers of each variables at the top and bottom 1% level.

Table 3 shows the reduced form results. We are interested in is testing whether shocks to own customers and suppliers have an impact, above and beyond the effects of the shocks to all potential customers and suppliers. Focusing on the coefficients on the trade shocks that firms' customers and suppliers received, the results in Table 3 are suggestive of the transmissions

[13] They vary at the firm i - sector u level, as we exclude firm i's own exports and imports if firm i is producing sector u good. For the weights that firm i assigns to each firm within a sector, we use the firms' sales to domestic final demand as corresponding weights. Therefore we have

$$X_{ut,t-1}^{IV,-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} X_{jt,t-1}^{IV}$$
$$M_{ut,t-1}^{IV,-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} M_{jt,t-1}^{IV}$$
$$X_{ut-1,t-1}^{IV,-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} X_{jt-1,t-1}^{IV}$$
$$M_{ut-1,t-1}^{IV,-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} M_{jt-1,t-1}^{IV},$$

where U_t is the set of firms producing sector u good at t, and V_{iHt} is firm i's sales to domestic final demand at t.

Table 3: Reduced form results

	(1) $\Delta\ln$ Total Sales	(2) $\Delta\ln$ Dom Sales	(3) $\Delta\ln$ Labor Cost	(4) $\Delta\ln$ Dom Network Inputs
$\Delta\ln X_i^{IV,C}$	0.132***	0.130***	0.0203	0.111***
	(0.0118)	(0.0196)	(0.0139)	(0.0191)
$\Delta\ln M_i^{IV,S}$	0.0191	0.0593**	-0.0407**	0.114***
	(0.0159)	(0.0262)	(0.0187)	(0.0257)
$\Delta\ln X_i^{IV,TC}$	0.0253***	0.0390***	0.0361***	0.0219***
	(0.00300)	(0.00503)	(0.00353)	(0.00484)
$\Delta\ln M_i^{IV,TS}$	0.0500***	0.0198***	0.0748***	0.190***
	(0.00446)	(0.00735)	(0.00522)	(0.00722)
$\Delta\ln X_i^{IV}$	0.0966***	0.0295**	0.0137	0.0852*
	(0.00796)	(0.0131)	(0.00939)	(0.0129)
$\Delta\ln M_i^{IV}$	0.163***	0.113***	0.0710***	0.129***
	(0.0107)	(0.0176)	(0.0126)	(0.0173)
N	84632	83674	85178	84921

Notes: Standard errors in parentheses. All variables are in terms of yearly log differences from 2002 to 2012. All specifications include year fixed effects.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

of trade shocks through production linkages. Controlling for shocks that potential customers have received, positive demand shock on a firm's actual customers' leads to an expansion of the firm and increases its domestic sales and domestic network inputs, along its total sales. On the other hand, when a firm's suppliers increase their imports, this translates to larger input purchases of the firm, and also to an increase in domestic sales.

Interestingly, a trade shock affecting postively firm imports leads not only to an increase in firm's domestic network inputs, but also in the firm's expenditure on labor. These findings are consistent with the restriction in the model that $\sigma > \rho$, which implies that domestic goods, labor and foreign sourcing are complements at the firm level. However, the complementarity between labor and domestic goods seems to be lower, as we find a reduction in the expenditure on labor when the firm's suppliers experience exogenous shocks that increase their imports.

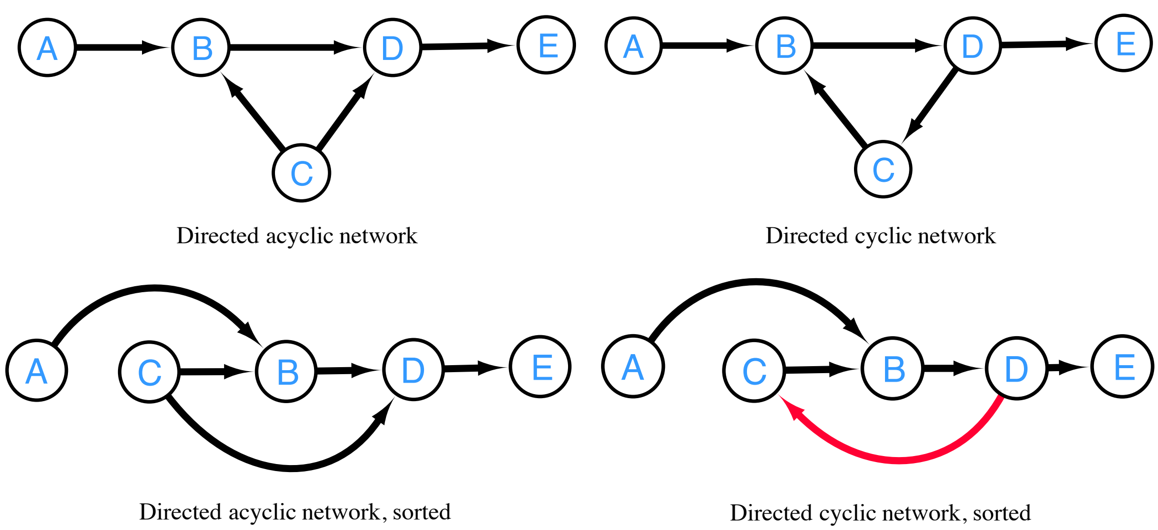
We run the reduced form regression with six instrumental variables as our main specification, but in Appendix E, we also show results for the two stage least squares with four endogenous variables: $\Delta\log X_{it}^C$, $\Delta\log M_{it}^S$, $\Delta\log X_{it}^{TC}$ and $\Delta\log M_{it}^{TS}$. Though the second stage coefficients with multiple endogenous variables are harder to interpret, overall we obtain similar results that suggest shocks transmit through firm-to-firm production linkages.

3.3 Shape of the network

As described in section 2.3.2, when we consider endogenous network formation, we order firms into a sequence and impose that a firm can only source from other firms that are positioned prior to the firm in the sequence.

By the construction of this mechanism, the resulting directed network will be acyclic. As shown in Figure 3, in an acyclic firm network, there exists at least one way to sort firms so that all directed edges face one direction. On the other hand, in a cyclic network of firms, at least one edge will face the opposite direction in any sorting. This feature of our network formation mechanism is admittedly restrictive. The real firm-to-firm network we see in the data is cyclic, and we do not have that feature in our model with an endogenous network formation.[14]

Figure 3: Examples of acyclic and cyclic networks



Therefore, in this section we quantify the degree of how cyclic the Belgian network is, to help us assess the plausibility of our network formation mechanism. Let $\nu(i)$ be an ordering of firms that maps firms $\{i, j, k, \cdots\} \in \Theta$ into numbers from $\{1, \cdots N\}$. We seek for the optimal $\nu(k)$ that minimizes the following objective function:

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\},$$

where Z_j is the supplier set of firm j. This problem essentially tries to find a sorting order of firms that minimizes the number of directed edges that are facing the opposite direction (i.e., violating the ordering), or the number of red arrows facing to the left, in the cyclic

[14]Note that we can easily incorporate a cyclic network in the model with a fixed network structure.

network in Figure 3.

To solve this problem, which is also known as the feedback arc set problem, we adopt an algorithm proposed by Eades, Lin and Smyth (1993). The details of the computational algorithm and implementation are presented in Appendix F. We obtain a local minimum, and under the attained sorting order around 17% of edges in the whole firm-to-firm network in 2012 violate the ordering. As this ordering is not a guaranteed global optimum, the Belgian firm-to-firm trade network has at most 17% of its edges in violation of an acyclic network.[15] We also search for an ordering that minimizes the value of firm-to-firm sales in violation of acyclicity. We have obtained an ordering under which at most 22% of firm-to-firm sales are in violation of acyclicity.

A natural question that arises is how different the structure of an economy with an acyclic network is in comparison to the economy observed in the data. Interestingly, the dropped links seem to be random in the following sense: We find that when calculating input-output tables with 72 sectors, the correlation between the input-output table coefficients from the full data and the data without links in violation of the ordering is 0.92 when using the the unweighted ordering algorithm output. The correlation is even higher, 0.97, when using the weighted ordering algorithm output.

We next turn to the structural analysis of the gains from trade for the Belgian economy.

4 Structural analysis

In this section, we start by providing a quantitative assessment of the implications of banning foreign inputs on the costs of Belgian firms and the price index faced by the Belgian consumers given a fixed network structure. This quantitative assessment can be conducted by directly using the observed firm-to-firm trade flows, international trade flows, and labor input information, as well as an estimate of the elasticity of substitution in the production and utility function. We then discuss the estimation of our model's parameters in order to simulate the model with endogenous network structure. Finally, in Section 4.5 we assess the robustness of our results obtained under fixed network structure in the model with endogenous network formation (in progress).

Throughout the analysis, we fix the elasticity of substitution in the utility function to a plausible parameter value and provide sensitivity analysis of results when altering that parameter value. In our baseline estimation, we set $\sigma = 4$. Similarly, we fix the elasticity of substitution in the production function to a baseline value of $\rho = 2$ and provide sensitivity

[15] While there is no perfect reference point for this figure, we can compare it to the structure of the directed social network Twitter. Simpson, Srinivasan and Thomo (2016) calculate that 23% of edges are in violation of acyclicity in the Twitter network in the year 2010.

analysis to altering that parameter.

We first turn to the results given fixed domestic production networks.

4.1 Results given fixed network

To assess the implications of banning foreign inputs, we compute the firm level cost increases by making use of proposition 1. We take the firm-to-firm network structure in 2012 and assume that the observed linkages and nominal wages are fixed. Proposition 1 tells us that the direct foreign input shares s_{Fj} and the total shares of foreign inputs s_{Fj}^{Total} for each firm directly translate to the cost changes that firms face from banning foreign inputs.

In addition to the economies where only direct shares or total shares of foreign inputs are considered, we also consider an economy with roundabout production in which each firm buys the same bundle of intermediate inputs (Blaum et al., 2016). We can derive an analogous equation for a firm's cost changes upon banning foreign inputs in this roundabout economy:[16]

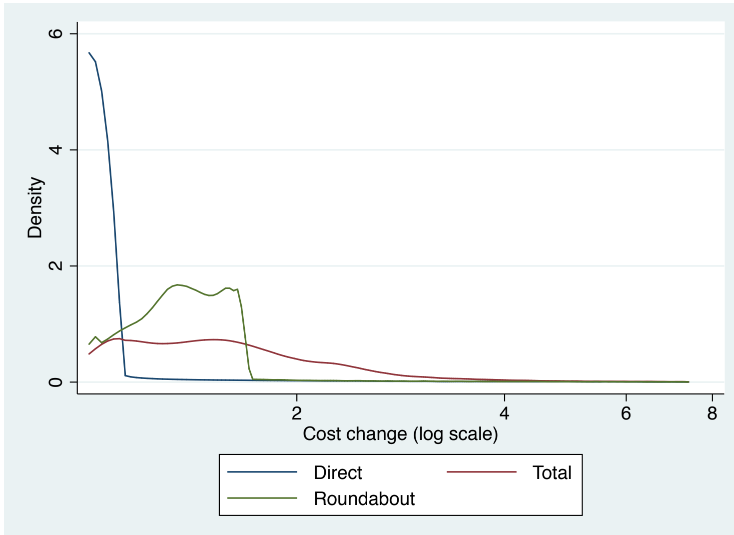
$$\left(\hat{c}_j \left|_{roundabout}^{p_{F\cdot}\to\infty}\right.\right)^{1-\rho} = s_{\ell j} + s_{Dj}\left(\sum_k s_{kD}\left(\hat{c}_k \left|_{roundabout}^{p_{F\cdot}\to\infty}\right.\right)^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}},$$

where s_{Dj} is the share of firm j's domestic intermediate good purchases and s_{kD} is the share of firm k in the intermediate good bundle (measured by firm k's share in total domestic final good sales). We obtain $\hat{c}_j \left|_{roundabout}^{p_{F\cdot}\to\infty}\right.$ by solving the above system of equations. We plot the distribution of the log cost changes from banning imports calculated under the three different measures in Figure 4.

One can clearly see that the cost increases we see when taking into account the full network are much more dispersed and higher in value than the cost increases we see when taking into account only the direct effect. Even compared to the cost increases calculated in a roundabout production economy, the distribution of cost increases is shifted to the right when taking into account the full production network. In the roundabout production economy, the non-importing firm's cost increase is bounded above by the price increase of the composite intermediate good. We see that when taking the actual production network into consideration, many firms have a cost increase above 65% = exp(.5) − 1, while the roundabout model would suggest that only very few importing firms have cost increases larger than 65%. In Appendix G, we plot the same distributions for different parameter values of σ and ρ. Even for alternative parameters, the cost change distribution one finds when taking into the account the full network tends have a much thicker right tail than that distribution under roundabout production.

[16] See Appendix A.2 for the derivation.

Figure 4: Distributions of $\hat{c}$ from banning imports



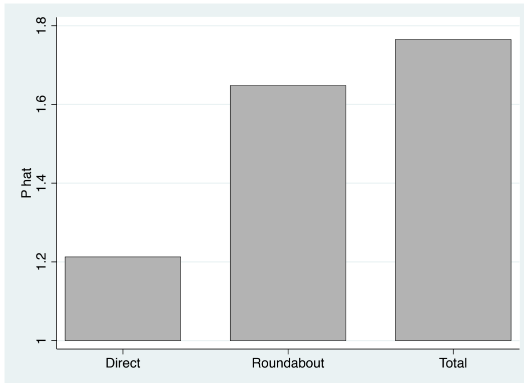
Notes: The parameters used are $\rho = 2, \sigma = 4$. See the appendix for derivations and plots for different parameter values.

Having computed the firms' cost changes across different economies, we make use of proposition 4 to analyze the implications for the aggregate consumer price index from banning foreign inputs. Figure 5 reports the values of changes in aggregate price indices, $\hat{P}$, for the three different cases. The increase in the price index is much larger when one takes into account the whole network structure, than when one only takes into account firms' direct exposure to foreign inputs. The price index is expected to rise by 21 percent if only the direct effect of banning imports is considered (pretending that all intermediate goods had only domestic content) and is expected to rise by 77 percent if the full network structure of domestic production is considered. Moreover, also compared to a model of roundabout production, we find a larger change in the price index allowing for firm-to-firm linkages. Under roundabout production, the domestic price index increases by 65 percent. In Appendix G, we report the analogous numbers for $\hat{P}$ under different parameters of σ and ρ.

Before discussing our results under the formation of an endogenous domestic production network, we note that the increase in the aggregate consumer price index from banning foreign inputs under an exogenous network is very similar if one only uses transactions that are consistent with the acyclic network obtained by the ordering algorithm described in Section 3.3. Specifically, we keep the direct import share of each firm the same as in the data, set all transactions in violation of the ordering to zero, and adjust all other domestic firm-to-firm input shares such that share of each firm j's input purchases, $\sum_{i \in Z_j} s_{ij}$, is unchanged.[17] The results presented in Table 17 in the Appendix show that the gains from trade under an

[17]The only exception is when there is no other domestic supplier of a firm, in which case in the data with only acyclic transactions the domestic firm-to-firm input share is set to zero.

Figure 5: Change in aggregate price index, $\hat{P}$, from banning imports



Notes: The parameters used are $\rho = 2, \sigma = 4$. See the appendix for derivations and plots for different parameter values.

exogenous network are virtually identical if the domestic production network is acyclic.

We proceed by discussing the estimation of our model with an endogenous network formation and revisit the same counterfactual in that model further below.

4.2 Estimation of model parameters given endogenous network

Below we present the estimation of our model with an endogenous network formation. We are currently finishing the full estimation with international trade, and present below the estimation and results of a purely domestic economy with an endogenous network which arises if the barriers to trade are infinite. For the remainder of the paper, we normalize $\alpha_{\ell j} = 1$ and $\beta_{jD} = 1$.

As a first step of the estimation, we recover the productivity distribution of firms (scaled by some general equilibrium objects) from the identity

$$\frac{x_{iH}}{s_{\ell i}^{(\sigma-1)/(1-\rho)}} = \phi_i^{\sigma-1} \frac{\mu^{1-\sigma} w_\ell^{1-\sigma}}{P^{1-\sigma}} E. \tag{23}$$

Observing all the terms on the LHS enables us to estimate the distribution $\phi_i^{\sigma-1} \frac{\mu^{1-\sigma} w_\ell^{1-\sigma}}{P^{1-\sigma}} E$. After visually inspecting the distribution, we assume that the distribution is log-normal, and estimate the scale parameter as -2.23 and the dispersion parameter as 1.31.

We next turn to the estimation of the parameters for the distribution of the firm-pair-specific shifter in the production function, α_{kj}, as well as the distribution of fixed cost parameters $w_\ell f_{jk}$. We again impose log-normality of the distributions and estimate the scale and dispersion parameters of the distributions. We assume that both α_{kj} and f_{kj} draws are i.i.d. across customer firms j and supplier firms k.

We use simulated methods of moments to estimate our parameters. The draws of the fixed costs govern the extensive margins of firm-to-firm trade. Thus, we use moments from firm level indegree distribution to identify Φ^{f}_{scale} and Φ^{f}_{disp}. To do this, we match the model to have the same quartile distribution of indegrees. In addition, we sort firms by total sales and match the indegree distributions for each sales quartile. Following the procedure used by Eaton, Kortum and Kramarz (2011), we include in the first vector of moments generated by the model, $\hat{m}_1(\delta)$, the proportion of firms that has a number of indegrees equal to the first, second, third, and fourth quartile in the data. We also include the same fraction indegrees for each sales quartile. This generates 20 elements in the vector $\hat{m}_1(\delta)$.

The draws of $\alpha_{kj}(j)$ govern the distribution of both the intensive margin and the extensive margin of firm-to-firm trade. To identify the parameters Φ^{α}_{scale} and Φ^{α}_{disp}, we include as a second set of moments statistics on the labor share of firms. Again, we aim to match the fraction of firms in the data that have labor shares in the first, second, third, and fourth quartile of the actual labor share in the data. Similarly to the first moment, we also include the same statictic separately for each sales quartile. This generates 20 elements in the vector $\hat{m}_2(\delta)$. Relatedly, we also aim to match the distribution of the actual firm-to-firm input shares (conditional on observing trade between firms). Here we just include the fraction of firms in the four quartiles of that distribution (using as thresholds the quartiles observed in the data). This generates 4 elements in the vector $\hat{m}_3(\delta)$.

We describe the difference between the moments in the data and in the simulated model by $\hat{y}(\delta)$:

$$\hat{y}(\delta) = m - \hat{m}(\delta) = \begin{bmatrix} m_1 - \hat{m}_1(\delta) \\ m_2 - \hat{m}_2(\delta) \\ m_3 - \hat{m}_3(\delta) \end{bmatrix},$$

and the following moment condition is assumed to hold at the true parameter value δ_0:

$$E\left[\hat{y}(\delta_0)\right] = 0. \tag{24}$$

The method of simulated moments selects the model parameters that minimize the following objective function:

$$\hat{\delta} = \arg\min_{\delta} \left[\hat{y}(\delta)\right]^{\top} \mathbf{W} \left[\hat{y}(\delta)\right], \tag{25}$$

where $\mathbf{W}$ is a weighting matrix.[18]

4.3 Estimation results

Table 4 shows the values of the estimated parameters.

[18] We weight the moments equally, hence the weighting matrix is the identity matrix.

Table 4: Estimated parameters

Φ^{α}_{scale}	Φ^{α}_{disp}	Φ^{f}_{scale}	Φ^{f}_{disp}
-3.75	2.49	-11.31	8.59

The parameter estimates of these distributional parameters are by themselves not very informative. We proceed by discussing the model fit given these parameter estimates.

4.4 Model fit

We first discuss the model's fit of moments it was targeted to fit. The results are in Table 5. Note that instead of showing the moments directly (i.e., the fraction of firms falling into each quartile bin), we show the values of the 25th, 50th, and 75th percentiles directly. The model does a pretty good job at fitting the targeted statistics. Most statistics are very close between the model and the data. The model comes a bit short off fitting the 75th percentile of the indegree distribution for the firms in the largest sales quartile (111 in data and 75 in the estimated model). Also, while the model has some difficulties matching the quartiles of the firm-to-firm trade shares, it matches the labor share quartiles quite well.

We next discuss the fit of some statistics of the network that the model was not directly targeted to fit in the estimation. Specifically, have we not targeted directly the association of size between buyers and sellers that trade with each other. Consistent with the data, the estimated model predicts that there is weakly negative assortative matching between buyers and sellers (see Table 6).

4.5 Counterfactuals with endogenous network structure

t.b.a.

5 Conclusion

t.b.a.

Table 5: Model fit: targeted moments

	Data	Model
Indegree 25th percentile	15	14
Indegree 50th percentile	28	29
Indegree 75th percentile	49	52
1st sales quartile: Indegree 25th percentile	7	8
1st sales quartile: Indegree 50th percentile	12	16
1st sales quartile: Indegree 75th percentile	18	29
2nd sales quartile: Indegree 25th percentile	16	14
2nd sales quartile: Indegree 50th percentile	24	28
2nd sales quartile: Indegree 75th percentile	32	48
3rd sales quartile: Indegree 25th percentile	26	19
3rd sales quartile: Indegree 50th percentile	38	36
3rd sales quartile: Indegree 75th percentile	50	60
4th sales quartile: Indegree 25th percentile	44	25
4th sales quartile: Indegree 50th percentile	70	46
4th sales quartile: Indegree 75th percentile	111	75
Share of labor costs 25th percentile	0.12	0.13
Share of labor costs 50th percentile	0.28	0.31
Share of labor costs 75th percentile	0.50	0.61
1st sales quartile: Share of labor costs 25th percentile	0.17	0.25
1st sales quartile: Share of labor costs 50th percentile	0.37	0.48
1st sales quartile: Share of labor costs 75th percentile	0.61	0.78
2nd sales quartile: Share of labor costs 25th percentile	0.17	0.14
2nd sales quartile: Share of labor costs 50th percentile	0.33	0.31
2nd sales quartile: Share of labor costs 75th percentile	0.51	0.60
3rd sales quartile: Share of labor costs 25th percentile	0.12	0.10
3rd sales quartile: Share of labor costs 50th percentile	0.27	0.24
3rd sales quartile: Share of labor costs 75th percentile	0.46	0.52
4th sales quartile: Share of labor costs 25th percentile	0.08	0.07
4th sales quartile: Share of labor costs 50th percentile	0.17	0.21
4th sales quartile: Share of labor costs 75th percentile	0.35	0.49
Firm-to-Firm share 25th percentile	0.0002	0.0000
Firm-to-Firm share 50th percentile	0.0015	0.0003
Firm-to-Firm share 75th percentile	0.0069	0.0028

Table 6: Model fit: non-targeted moments

	Data	Model
Corr (Indegree Buyer, Outdegree Seller)	-0.05	-0.10
Corr (Sales Buyer, Sales Seller)	-0.02	0.01

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A Theoretical Results

A.1 Proof of Proposition 1

Proof.

We have

$$\begin{aligned} s_{Fj}^{Total} &= s_{Fj} + \sum_i s_{ij} s_{Fi}^{Total} \\ &= s_{Fj} + \sum_i s_{ij} \left[s_{Fi} + \sum_k s_{ki} \left(s_{Fk} + \cdots \right) \right] \end{aligned}$$

and

$$c_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \mu^{1-\rho} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho} + \alpha_{Fj}^{\rho-1} \phi_j^{\rho-1} p_{Fj}^{1-\rho}.$$

If only considering the direct effect (i.e., assuming that suppliers' cost increases will not translate into price increases for their customers), with $\rho > 1$ and $p_{Fj} \to \infty$ for all j (i.e., autarky), the cost for firm j becomes

$$\bar{c}_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \mu^{1-\rho} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho}.$$

Therefore, the change in unit cost is

$$\begin{aligned} \hat{c}_j^{1-\rho} \Big|_{direct}^{p_{F\cdot} \to \infty} &= \frac{\bar{c}_j^{1-\rho}}{c_j^{1-\rho}} = 1 - s_{Fj} \\ &= s_{\ell j} + \sum_k s_{kj}. \end{aligned}$$

Now consider the indirect effects (i.e., assuming that suppliers' cost increases will translate into price increases for their customers). The unit cost after $p_{Fj} \to \infty$ for all j (assuming that the nominal wage, w_ℓ, does not change) is

$$\tilde{c}_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \mu^{1-\rho} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho},$$

thus

$$
\begin{aligned}
\hat{c}_j^{1-\rho} \mid_{total}^{p_F.\to\infty} &= \frac{\tilde{c}_j^{1-\rho}}{c_j^{1-\rho}} \\
&= \frac{\sum_k \alpha_{kj}^{\rho-1} \mu^{1-\rho} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho}}{c_j^{1-\rho}} \\
&= s_{\ell j} + \sum_k s_{kj} \hat{c}_k^{1-\rho} \mid_{total}^{p_F.\to\infty} \\
&= s_{\ell j} + \sum_k s_{kj} \left[s_{\ell k} + \sum_i s_{ik} \left(s_{\ell i} + \cdots \right) \right]
\end{aligned}
$$

We can observe that

$$\hat{c}_j^{1-\rho} \mid_{total}^{p_F.\to\infty} < \hat{c}_j^{1-\rho} \mid_{direct}^{p_F.\to\infty}$$

Also notice that

$$
\begin{aligned}
\hat{c}_j^{1-\rho} \mid_{total}^{p_F.\to\infty} &= s_{\ell j} + \sum_k s_{kj} \left[s_{\ell k} + \sum_i s_{ik} \left(s_{\ell i} + \cdots \right) \right] \\
&= \left(1 - s_{Fj} - \sum_k s_{kj} \right) + \sum_k s_{kj} \left[\left(1 - s_{Fk} - \sum_i s_{ik} \right) + \sum_i s_{ik} \left(\left(1 - s_{Fi} + \sum_l s_{li} \right) + \cdots \right) \right] \\
&= 1 - \left(s_{Fj} + \sum_k s_{kj} \left[s_{Fk} + \sum_i s_{ik} - \sum_i s_{ik} \left(1 - s_{Fi} + \sum_l s_{li} + \cdots \right) \right] \right) \\
&= 1 - \left(s_{Fj} + \sum_k s_{kj} \left[s_{Fk} + \sum_i s_{ik} \left(s_{Fi} + \cdots \right) \right] \right) \\
&= 1 - s_{Fj}^{Total}
\end{aligned}
$$

Therefore, firms' change in unit costs upon $p_f \to \infty$ when considering only direct effects and when considering full network effects are as follows:

$$
\begin{aligned}
\hat{c}_j^{1-\rho} \mid_{direct}^{p_F.\to\infty} &= \left(1 - s_{Fj} \right)^{\frac{1}{1-\rho}} \\
\hat{c}_j^{1-\rho} \mid_{total}^{p_F.\to\infty} &= \left(1 - s_{Fj}^{Total} \right)^{\frac{1}{1-\rho}}
\end{aligned}
$$

■

A.2 Cost reduction under roundabout production

Firm j produces its goods with a CES production technology, using domestic intermediate goods, foreign imports, and labor. The implied unit cost of firm j becomes

$$c_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} P_D^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}\right)^{\frac{1}{1-\rho}},$$

where P_D is a price index of domestic intermediate goods. Associated input shares are $s_{Dj} = \frac{\phi_j^{\rho-1}\alpha_{Dj}^{\rho-1}P_D^{1-\rho}}{c_j^{1-\rho}}$, $s_{Fj} = \frac{\phi_j^{\rho-1}\alpha_{Fj}^{\rho-1}p_{Fj}^{1-\rho}}{c_j^{1-\rho}}$, and $s_{\ell j} = \frac{\phi_j^{\rho-1}\alpha_{\ell j}^{\rho-1}w_\ell^{1-\rho}}{c_j^{1-\rho}}$.

As in Blaum et al. (2016), we let domestic intermediate goods be produced via roundabout production, with CES substitution parameter σ. The price of an intermediate good is therefore equal to the CES price index,

$$P_D = \left(\sum_j \alpha_{jD}^{\sigma-1} p_{jD}^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

where p_{jD} is the price that firm j charges in the aggregation process. Let $p_{jD} = \mu_{jD} c_j$, where μ_{jD} is a constant. We can additionally define $s_{jD} = \frac{\alpha_{jD}^{\sigma-1} p_{jD}^{1-\sigma}}{P_D^{1-\sigma}}$, which is the firm j's contribution to the intermediate good. We use the firm's share of domestic sales.

Consider a change in c_j, upon $p_{Fj} \to \infty$ for all j.

$$\begin{aligned} c_j &= \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} P_D^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}\right)^{\frac{1}{1-\rho}} \\ \tilde{c}_j &= \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} \tilde{P}_D^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}\right)^{\frac{1}{1-\rho}} \\ \tilde{P}_D^{1-\sigma} &= \sum_j \alpha_{jD}^{\sigma-1} \mu_{jD}^{1-\sigma} \tilde{c}_j^{1-\sigma} \end{aligned}$$

Combining these,

$$\tilde{c}_j^{1-\rho} = \phi_j^{\rho-1} \alpha_{Dj}^{\rho-1} \left(\sum_j \alpha_{jD}^{\sigma-1} \mu_{jD}^{1-\sigma} \tilde{c}_j^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}} + \phi_j^{\rho-1} \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}$$

then

$$
\begin{aligned}
\hat{c}_j^{1-\rho}\,|_{roundabout}^{p_F.\to\infty} &= \frac{\phi_j^{\rho-1}\alpha_{Dj}^{\rho-1}\left(\sum_j \alpha_{jD}^{\sigma-1}\mu_{jD}^{1-\sigma}\tilde{c}_j^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}} + \phi_j^{\rho-1}\alpha_{\ell j}^{\rho-1}w_\ell^{1-\rho}}{c_j^{1-\rho}} \\
&= s_{\ell j} + s_{Dj}\frac{\left(\sum_j \alpha_{jD}^{\sigma-1}\mu_{jD}^{1-\sigma}\tilde{c}_j^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}}}{P_D^{1-\rho}} \\
&= s_{\ell j} + s_{Dj}\left(\frac{\sum_j \alpha_{jD}^{\sigma-1}\mu_{jD}^{1-\sigma}c_j^{1-\sigma}\hat{c}_j^{1-\sigma}\,|_{roundabout}^{p_F.\to\infty}}{P_D^{1-\sigma}}\right)^{\frac{1-\rho}{1-\sigma}} \\
&= s_{\ell j} + s_{Dj}\left(\sum_j s_{jD}\hat{c}_j^{1-\sigma}\,|_{roundabout}^{p_F.\to\infty}\right)^{\frac{1-\rho}{1-\sigma}}.
\end{aligned}
$$

The solution to this system of equations $\hat{c}_j^{1-\rho}\,|_{roundabout}^{p_F.\to\infty}$ is the change in unit costs of each firm, upon autarky.

A.3 Proof of Lemma 3

Rearranging equation (5), we obtain

$$
c(j)^{1-\rho} = \sum_k \phi_j^{\rho-1}\alpha_k(j)^{\rho-1}c(k)^{1-\rho} + \phi_j^{\rho-1}\alpha_\ell(j)^{\rho-1}w_\ell^{1-\rho}.
$$

In matrix form, this equation becomes

$$
\boldsymbol{c}^{1-\rho} = \left(I - A'\right)^{-1}\boldsymbol{\phi}^{\rho-1}\circ\boldsymbol{\alpha}_\ell^{\rho-1}\circ w_\ell^{1-\rho},
$$

where the (i,j) element of A is $\phi_j^{\rho-1}\alpha_i(j)^{\rho-1}$. The assumption that the matrix $\left(I - A'\right)$ is invertible guarantees that there is a unique vector $\boldsymbol{c}$ that solves the equation above. With the cost vector $\boldsymbol{c}$ and constant mark-ups in sales to final consumers, one can compute the aggregate price index P according to (3).

A.4 Proof of Proposition 4

Denote post-shock equilibrium variable x with $\tilde{x}$. From equation (3), we have the expression for the price index after the shock,

$$
\tilde{P} = \left(\sum_i \beta_i^{\sigma-1}\mu\tilde{c}_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.
$$

Combining this expression with the pre-shock price index P, we have

$$
\begin{aligned}
\hat{P} &= \frac{\tilde{P}}{P} \\
&= \left(\frac{\sum_i \beta_i^{\sigma-1} \mu \tilde{c}_i^{1-\sigma}}{P^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \\
&= \left(\sum_i \frac{\beta_i^{\sigma-1} \mu c_i^{1-\sigma}}{P^{1-\sigma}} \hat{c}_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
&= \left(\sum_i s_{iH} \hat{c}_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}
\end{aligned}
$$

where s_{iH} denotes firm i's share in final consumption. This equation says that the change in the aggregate price index depends on each firm's change in cost and its share in final consumption before the shock.

B Algorithm for network formation

We assume that when a firm sells its good to other domestic firms, the bargaining game is such that the customer firm takes all the surplus and the supplier firm receives no profits. Firms generate profits only from their sales to final demand.

We simulate endogenous networks, given the distribution of ϕ_i, $G(\phi)$ and parameters. Among the parameters we have the maximum number of potential suppliers that a firm can have, $\bar{Z}$.

We follow the steps below to simulate the firm-to-firm trade network formation.

1. Firms with productivities ϕ_i are randomly sorted, and indexed with $i = 1, 2, 3, \cdots$.

2. All firms make a common guess of aggregate demand term: $B = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} EP^{\sigma-1}$ and wage level w.

3. We assume that firms decide on their sourcing strategies in sequence of i. Firm 1 decides its sourcing strategy and determines c_1, then firm 2 decides its sourcing strategy and determines c_2, and so on. When firms make their sourcing decisions, we assume that all firms are able to use labor, but firm i is only able to choose its suppliers from its potential supplier set $\bar{Z}_i$. $\bar{Z}_i$ is defined as the union of the set of foreign countries and the set of firms $\left\{\max\left\{1, i-\bar{Z}\right\}, \cdots, i-1\right\}$. We determine which countries and which firms among $\bar{Z}_i$ that firm i sources from, using the Jia algorithm, and compute c_i. After the final firm $i = N$ decides its sourcing strategy, the whole vector $\boldsymbol{c}$ and the supplier sets of all firms $\boldsymbol{Z}$ are determined.

4. Given the network $\boldsymbol{Z}$, B, and w, we are able to compute the equilibrium variables.

    (a) Sales to final demand of firm i is computed by $X_{iH} = \sigma B c_i^{1-\sigma}$.

    (b) The cost of inputs used for firm i's sales to final demand is thus $C_{iH} = \frac{\sigma-1}{\sigma} X_{iH}$.

    (c) The total input costs of firms, C_i are calculated by solving the system of linear equations below:

    $$C_i = C_{iH} + \sum_j s_{ij} C_j,$$
    $$\rightarrow \mathbf{C} = (I - S)^{-1} \mathbf{C}_H$$

    where $\mathbf{C}$ and $\mathbf{C}_H$ are vectors of C_i and C_{iH}, and the i, j element of matrix S is s_{ij}.

    (d) The total sales of firm i is then $X_i = X_{iH} + C_i - C_{iH}$.

5. Given the above variables, we can update the guess of B, and iterate over steps 2 to 4.

    (a) Compute new guess $E^{'}$ using equation (16).

    (b) Compute new guess $P^{'}$ using $\boldsymbol{c}$.

    (c) Compute new guess $w^{'}$ using equation (18).

C Data Appendix

C.1 Grouping VAT-ids into firms

As mentioned in the main text, all our datasets are recorded at the VAT-id level. We utilize ownership filings in the Annual Accounts and the Foreign Direct Investment data in order to aggregate multiple VAT-ids into firms.

We group all VAT-ids into firms if they are linked with more than or equal to 50% of ownership. In addition, we group all VAT-ids into firms if they share the same foreign parent firm that holds more than or equal to 50% of their shares. We use a "fuzzy string matching" method to determine whether they share the same foreign parent firm, by obtaining similarity measures of all possible pairs of foreign firms' names. Lastly, in order to correct for misreportings, we also add links to the VAT-id pairs if the two were linked one year before and one year after. We define a firm as the group of VAT-ids that are directly and indirectly linked.

Given these groupings of VAT-ids, we then choose the "most representative" VAT-id for each firm. We use this "head VAT-id" as the identifier of the firm.[19] Then, in order to make the identifiers consistent over time, we make the following adjustment. We take firms whose head VAT-id was not an identifier of any firm in the previous year. For such firms, if there exists a VAT-id within the firm which was a head VAT-id in the previous year, then we switch the firm identifier to that former head VAT-id.[20]

Having determined the head VAT-id for each multiple VAT-id firm, we aggregate all the variables up to the firm level. For variables such as total sales and inputs, we adjust the aggregated variables with the amount of B2B trade that occurred within the firm, correcting for double counting. For other non-numeric variables such as firms' primary sector, we take the value of its head VAT-id.

C.2 Firm selection

Table 7 displays the same numbers for table 2, with statistics for all Belgian firms added.

[19]The criteria for determining the head VAT-id is as follows. (i) If there is only one VAT-id in the firm that filed all the full annual accounts, the VAT declarations, and the B2B filings, then this VAT-id is chosen as the head. (ii) If there are no such VAT-ids or multiple of them, then we choose the VAT-id that has the largest total assets reported. (iii) If there are no VAT-id that filed the annual accounts, then we choose the VAT-id that has the largest amount of total inputs, which is the sum of labor costs, B2B inputs, and imports.

[20]If there are multiple such VAT-id, then we choose the "most representative" VAT-id, using the same criteria as above.

Table 7: Coverage of all Belgian firms and selected sample

Year	GDP	Output	Imports	Exports	All Belgian Firms					Selected sample				
					Count	V.A.	Sales	Imports	Exports	Count	V.A.	Sales	Imports	Exports
2002	275	556	210	229	714,469	210	557	204	217	122,460	123	516	179	189
2007	345	715	300	314	782,006	274	770	294	282	136,370	157	713	280	269
2012	387	823	342	347	860,373	300	864	320	317	139,605	170	786	296	295

Notes: All numbers except for Count are denominated in billion Euro in current prices. Data for Belgian GDP, output, imports and exports are from Eurostat.

C.3 Reporting thresholds of the international trade data set

There are different reporting thresholds for the international trade data set, depending on if the trade occurred with an extra-EU country or within the EU. The data set covers all extra-EU exports and imports by firms with values higher than 1,000 Euro or with weights bigger than 1,000kg. Nevertheless, we also observe values less than 1,000 Euro as more firms use electronic reporting procedures. For intra-EU trade prior to 2006, the data set covers all exports and imports by firms whose combined imports from intra-EU countries that are more than 250,000 Euro a year. For intra-EU trade from 2006 onward, the thresholds for exports and imports changed to 1,000,000 Euro and 400,000 Euro, respectively. Import reporting thresholds became 700,000 Euro per year in 2010. While these reporting threshold for intra-EU trade imply we miss some trade transaction, they are set to capture at least 93% of aggregate Belgian trade in the micro-data, hence our data still contains the overwhelming majority of the value of Belgian trade.

C.4 Mapping CN codes into NACE codes

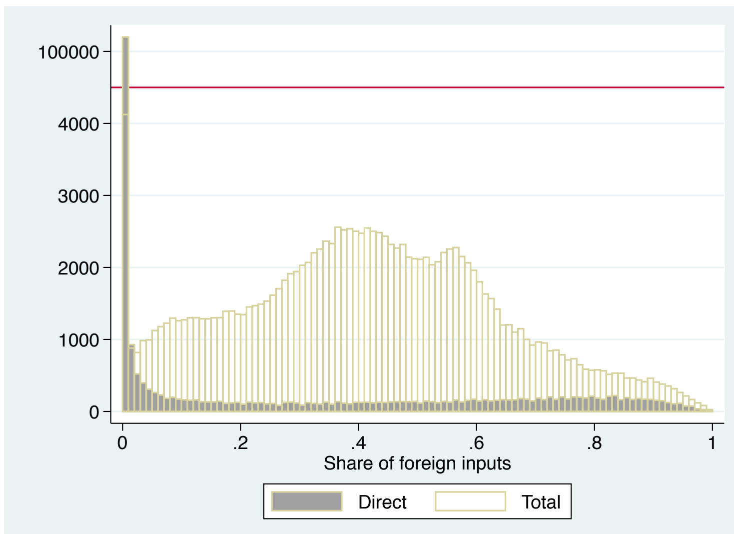
Our international trade data set records products in Combined Nomenclature (CN) codes, up to 8 digits. On the other hand, all other data sets that we use record the enterprise's primary sector in NACE Rev.2 code. To concord the two classifications, we convert the CN 8 digit codes into NACE Rev.2 codes. As the first 6 digits of CN codes are identical to the contemporary Harmonized System (HS) codes, we first convert those HS 6-digit codes to Classification of Products by Activity (CPA) codes. We then convert CPA codes to NACE codes, using the fact that CPA 2008 codes are identical to NACE Rev.2 codes up to 4 digits. This conversion allows us to convert more than 98% of all international trade recorded in our data set, in terms of values (in 2012).

D Descriptive statistics

D.1 Direct and Total foreign input shares

In Figures 6 and 7 we present both the direct and total foreign input shares first for the entire sample of private sector firms in Belgium and then differentiated by major sector.

Figure 6: Histogram of direct and total share of foreign inputs



Notes: Total share of foreign inputs of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct share of foreign inputs, and s_{ji} is j's share among i's inputs. The figure is based on the analysis of 139,605 private sector firms in Belgium in 2012.

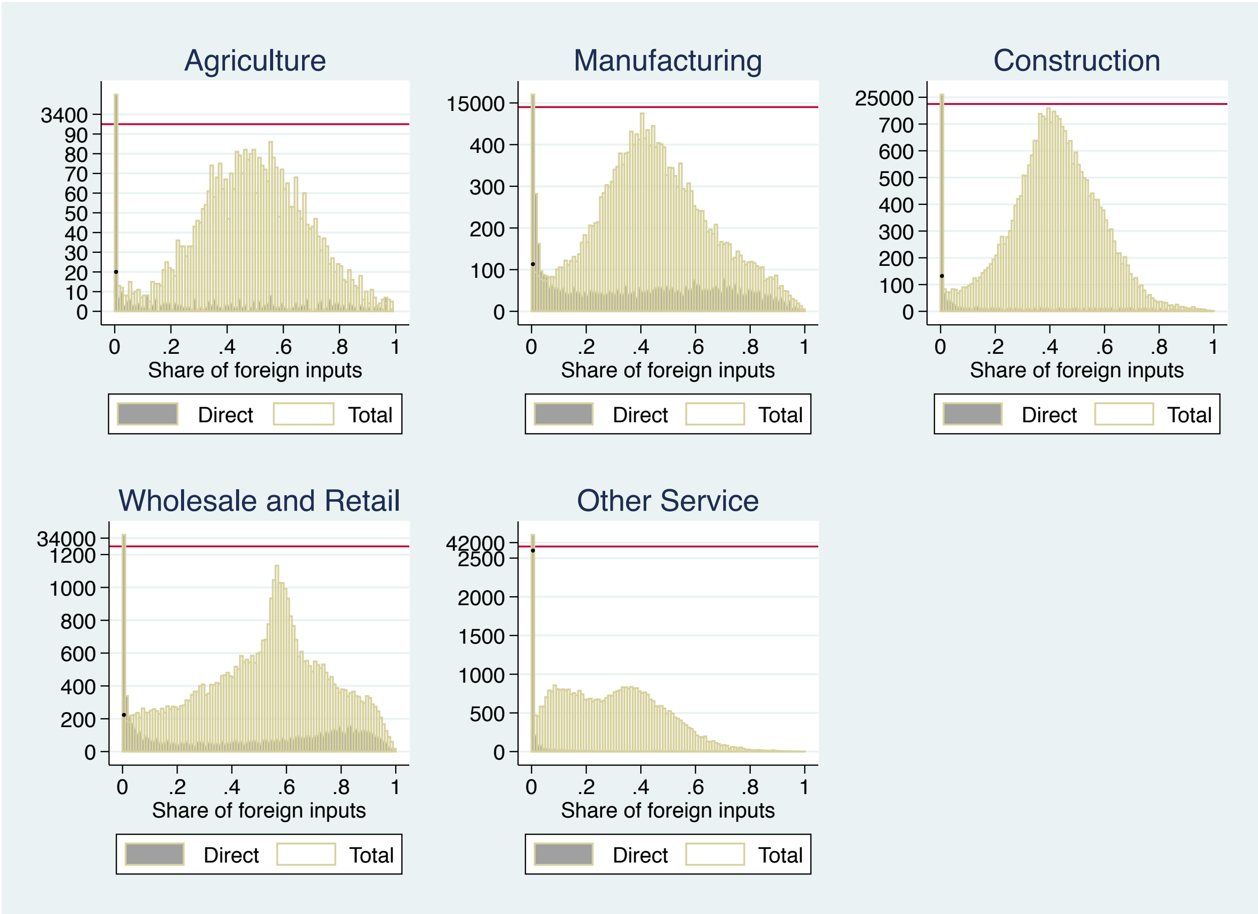
We summarize statistics on the distribution of the the direct and total share of foreign inputs by firm's industry in Table 8.

Table 8: Distribution of direct and total share of foreign inputs by firms' industry

Industry	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture	0.03	0.27	0	0.49	0.68	0.49
Construction	0.01	0.11	0	0.42	0.48	0.42
Manufacturing	0.11	0.59	0	0.45	0.75	0.44
Wholesale and Retail	0.10	0.42	0	0.52	0.75	0.55
Other Services	0.01	0.18	0	0.29	0.41	0.28

Notes: The numbers for the weighted mean is calculated using total input purchases of firms as the weight.

Figure 7: Histogram of direct and total share of foreign inputs by firms' industry



Notes: The black dot indicates the ending of the bar for the total share of foreign inputs. Total share of foreign inputs of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct share of foreign inputs, and s_{ji} is j's share among i's inputs. The figure is based on the analysis of 139,605 private sector firms in Belgium in 2012.

D.2 Industrial composition

Table 9 shows the industrial composition of our selected sample. Values for value added and output are in billion Euro.

D.3 Link survival

Table 9: Industrial composition in 2012

Industry	Count	V.A.	Output	Imports	Exports
Agriculture	3,704	1.49	10.8	1.71	2.26
Construction	26,364	18.3	54.6	5.00	3.65
Manufacturing	20,385	55.5	273	147	194
Wholesale and Retail	42,999	31.8	249	85.3	54.5
Other Services	43,495	50.3	130	17.6	17.0
Other	2,658	12.7	68.6	39.8	24.3
Total	139,605	170	786	296	295

Table 10: 2002 Link Survival

Occurred In...	Count	Col %	Cum %
2002	3,570,077	65.5	65.5
2002 & 2007	912,028	16.7	82.2
2002 & 2012	191,566	3.5	85.7
2002 & 2007 & 2012	778,734	14.3	100.0

E Two stage least squares

In the main text we presented the results from reduced form regressions. In this section we run two stage least squares, and see if similar results hold. Our focus here is still on shock transmission through the production network, thus we keep the instrumental variables for firms' own trade shocks and run with four endogenous variables: $\Delta \log X_{it}^C$, $\Delta \log M_{it}^S$, $\Delta \log X_{it}^{TC}$ and $\Delta \log M_{it}^{TS}$.

The regression equation is the following:

$$\begin{aligned}\Delta \log Y_{it} = &\beta + \beta_X^C \Delta \log X_{it}^C + \beta_M^S \Delta \log M_{it}^S \\ &+ \beta_X^{TC} \Delta \log X_{it}^{TC} + \beta_M^{TS} \Delta \log M_{it}^{TS} \\ &+ \beta_X \Delta \log X_{it}^{IV} + \beta_M \Delta \log M_{it}^{IV} + \varphi_t + \epsilon_{it}. \end{aligned} \tag{26}$$

$\Delta \log X_{it}^C$ and $\Delta \log M_{it}^S$ are the changes in average exports and imports of i's customers and suppliers:

$$\begin{aligned}\Delta \log X_{it}^C &= \log \sum_j \frac{\text{Sales}_{ijt}}{\text{TotalSales}_{it}} X_{jt} - \log \sum_j \frac{\text{Sales}_{ijt-1}}{\text{TotalSales}_{it-1}} X_{jt-1} \\ \Delta \log M_{it}^S &= \log \sum_k \frac{\text{Sales}_{kit}}{\text{TotalInputs}_{it}} M_{kt} - \log \sum_k \frac{\text{Sales}_{kit-1}}{\text{TotalInputs}_{it-1}} M_{kt-1}.\end{aligned}$$

$\Delta \log X_{it}^{TC}$ and $\Delta \log M_{it}^{TS}$ are the change in average exports and imports of i's potential customers and suppliers:

$$\Delta \log X_{it}^{TC} = \log \sum_u \tilde{s}_{iut} X_{ut}^{-i} - \log \sum_u \tilde{s}_{iut-1} X_{ut-1}^{-i}$$
$$\Delta \log M_{it}^{TS} = \log \sum_u s_{uit} M_{ut}^{-i} - \log \sum_u s_{uit-1} M_{ut-1}^{-i}$$

where

$$X_{ut}^{-i} = \sum_{j \in U_t, j \neq i} \frac{V_{jHt}}{\sum_{k \in U_t, k \neq i} V_{kHt}} X_{jt}$$
$$M_{ut}^{-i} = \sum_{j \in U_t, j \neq i} \frac{V_{jHt}}{\sum_{k \in U_t, k \neq i} V_{kHt}} M_{jt}.$$

First, Table 11 shows the OLS results. The first four columns are the results corresponding to equation (26), and the second four columns are the OLS where we have changes in firm's own exports and imports instead of their instruments.

Table 12 shows the first stage results for equation (26), for the specification where the LHS variable is the change in total sales. One would expect positive and significant coefficients on each respective instrumental variables. However, we find negative and significant coefficients on $\Delta \ln M_i^{IV,S}$ when the LHS variable is $\Delta \ln M_i^S$. We still find positive correlation of 0.24 for $\Delta \ln M_i^S$ and $\Delta \ln M_i^{IV,S}$, and with the SW F statistic close to its critical value for maximal IV relative bias lower than 10%, we conclude that the four instrumental variables jointly predict the endogenous variable $\Delta \ln M_i^S$.

Finally we present in table 13 the results for the second stage. Similarly to the reduced form regressions, here we also find that trade shocks transmit through production linkages. Controlling for shocks that potential customers have received, an exogenous increase in exports of a firm's actual customers leads to an expansion of the firm. In addition, when a firm's suppliers exogenously increase their imports, this also translates to increase in firms' sales and domestic network inputs. The magnitude of the 2sls coefficients is considerably larger than the magnitude of the OLS and reduced form coefficients. A comparison is complicated by the fact that the OLS coefficients are affected by endogeneity concerns and the generaly difficulty in interpreting IV coefficients with multiple endogenous variables.

Table 11: OLS regressions

	(1) $\Delta \ln$ Total Sales	(2) $\Delta \ln$ Dom Sales	(3) $\Delta \ln$ Labor Cost	(4) $\Delta \ln$ Dom Network Inputs	(1) $\Delta \ln$ Total Sales	(2) $\Delta \ln$ Dom Sales	(3) $\Delta \ln$ Labor Cost	(4) $\Delta \ln$ Dom Network Inputs
$\Delta \ln X_i^C$	0.00237***	0.0172***	0.00498***	0.00367***	0.00149***	0.0189***	0.00327***	0.00325***
	(0.000538)	(0.000891)	(0.000634)	(0.000865)	(0.000547)	(0.000992)	(0.000641)	(0.000929)
$\Delta \ln M_i^S$	-0.000918	0.000945	0.00650***	0.0168***	0.00208**	0.00106	0.00789***	0.0239***
	(0.000887)	(0.00145)	(0.00105)	(0.00144)	(0.000903)	(0.000903)	(0.00162)	(0.00154)
$\Delta \ln X_i^{TC}$	-0.00352***	-0.0155***	-0.000174	0.00140	-0.00309***	0.0175***	-0.0000154	0.00105
	(0.000638)	(0.00106)	(0.000751)	(0.00103)	(0.000638)	(0.00116)	(0.000747)	(0.00108)
$\Delta \ln M_i^{TS}$	-0.0124***	-0.00758***	-0.00915***	0.0400***	-0.000344	0.0000356	0.00281**	0.0550***
	(0.00112)	(0.00184)	(0.00132)	(0.00181)	(0.00114)	(0.00204)	(0.00134)	(0.00194)
$\Delta \ln X_i^{IV}$	0.114***	0.0458***	0.0179*	0.107***				
	(0.00785)	(0.0129)	(0.00927)	(0.0127)				
$\Delta \ln M_i^{IV}$	0.189***	0.138***	0.0739***	0.171***				
	(0.010)	(0.0169)	(0.0122)	(0.0166)				
$\Delta \ln X_i$					0.0459***	-0.0587***	0.0300***	0.0430***
					(0.000865)	(0.00155)	(0.00101)	(0.00147)
$\Delta \ln M_i$					0.0809***	0.0618***	0.0532***	0.0764***
					(0.00115)	(0.00206)	(0.00134)	(0.00194)
N	84632	83674	85178	84921	70346	69406	70678	70475
R^2	0.036	0.040	0.0047	0.043	0.14	0.067	0.044	0.086

Notes: Standard errors in parentheses. All variables are in terms of yearly log differences from 2002 to 2012. All specifications include year fixed effects.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 12: First stage regressions

	(1) $\Delta \ln X_i^C$	(2) $\Delta \ln M_i^S$	(3) $\Delta \ln X_i^{TC}$	(4) $\Delta \ln M_i^{TS}$
$\Delta \ln X_i^{IV,C}$	0.326***	0.0666	-0.0550	0.0923**
	(0.0860)	(0.0545)	(0.0727)	(0.0434)
$\Delta \ln M_i^{IV,S}$	0.106	-0.933***	0.0684	-0.746***
	(0.116)	(0.0732)	(0.0977)	(0.0582)
$\Delta \ln X_i^{IV,TC}$	0.215***	-0.0340**	0.290***	-0.0113
	(0.0218)	(0.0138)	(0.0184)	(0.0110)
$\Delta \ln M_i^{IV,TS}$	-0.0262	0.153***	0.0126	0.0674***
	(0.0325)	(0.0206)	(0.0274)	(0.0164)
$\Delta \ln X_i^{IV}$	0.0199	0.106***	0.0528	0.0464
	(0.0579)	(0.0367)	(0.0489)	(0.0292)
$\Delta \ln M_i^{IV}$	0.217***	0.204***	0.0630	0.130***
	(0.0776)	(0.0492)	(0.0656)	(0.0391)
N	84632	84632	84632	84632
SW F stat	13.81	9.81	17.28	9.86

Notes: Standard errors in parentheses. All variables are in terms of yearly log differences from 2002 to 2012. All specifications include year fixed effects.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 13: Second stage regressions

	(1) $\Delta \ln$ Total Sales	(2) $\Delta \ln$ Dom Sales	(3) $\Delta \ln$ Labor Cost	(4) $\Delta \ln$ Dom Network Inputs
$\Delta \ln X_i^C$	0.501***	0.423***	0.192	0.745*
	(0.186)	(0.123)	(0.135)	(0.443)
$\Delta \ln M_i^S$	0.929**	0.488**	0.889***	2.73***
	(0.379)	(0.248)	(0.243)	(0.806)
$\Delta \ln X_i^{TC}$	-0.219	-0.136	0.0278	-0.333
	(0.143)	(0.100)	(0.112)	(0.359)
$\Delta \ln M_i^{TS}$	-1.14**	-0.664**	-1.02***	-3.49***
	(0.505)	(0.337)	(0.329)	(1.08)
$\Delta \ln X_i^{IV}$	0.0525	0.009	-0.0337	-0.0539
	(0.0464)	(0.0329)	(0.0345)	(0.114)
$\Delta \ln M_i^{IV}$	0.0263	0.001	-0.0188	-0.103
	(0.075)	(0.0559)	(0.0567)	(0.179)
N	84632	83674	85178	84921

Notes: Standard errors in parentheses. All variables are in terms of yearly log differences from 2002 to 2012. All specifications include year fixed effects. Firms' imports and exports are instrumented using world export supply and world import demand. Customers' and suppliers' IV variables are aggregated using initial period's input and output shares.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

F Ordering algorithm

In this section we describe the implementation of the ordering algorithm to solve the feedback arc set problem. We begin by defining some terms and notation.

F.1 Terms and notation

- *graph / network*, $G = (V, E)$ - A collection of a set of edges E and set of vertices V. Edges describe the relationship between vertices. Two basic classifications of graphs are based on whether the edges are *directed* or *undirected* and whether they are *weighted* or *unweighted*
- $n = |V|$, $m = |E|$
- *cycle* - A path within a graph where a vertex is reachable from itself
- $d^+(u)$ - For a vertex $u \in V$ in a directed graph, number of outgoing edges
- $d^-(u)$ - For a vertex $u \in V$ in a directed graph, number of incoming edges

- $w^+(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of outgoing edges
- $w^-(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of incoming edges
- *sink* - A vertex $u \in V$ in a directed graph with $d^+(u) = 0$
- *source* - A vertex $u \in V$ in a directed graph with $d^-(u) = 0$
- *feedback arc set* - A set of edges from a directed cyclic graph that when removed make the graph acyclic
- $s = s_{left}s_{right}$ - Given 2 finite sequences s_{left} and s_{right} with the indicated notation we symbolize the *concatenation* operation. For example, if $s_{left} = (A, B, C)$ and $s_{right} = (X, Y, Z)$, then $s = s_{left}s_{right} = (A, B, C, X, Y, Z)$
- $\lfloor x \rfloor$ is the greatest integer less than or equal to x

F.2 Overview

The Belgian B2B data describes a weighted directed graph $G = (V, E)$. Vertices are firms and edges are sales between firms. The goal of the ordering algorithm is to order firms in a way such that a given firm only sells to firms further along in the ordering and only buys from firms that precede it. The condition desired by this ordering is known in graph theory as a *topological ordering* (Black (1999)). A topological ordering exists if and only if a graph is directed and acyclic. The B2B data is cyclic. For the *unweighted* case our motivation is to find a feedback arc set of minimal cardinality, that is, what is the minimum number of transactions that we need to drop (i.e., the "violators") from our network to satisfy our ordering condition? For the *weighted case*, we seek to find a feedback arc set such that the cumulative weight of the violating transactions is minimized. Finding a minimum feedback arc set is computationally difficult but approximation algorithms exist.

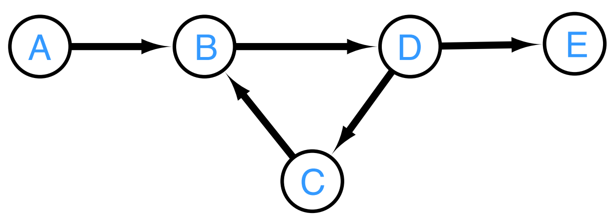
F.3 Unweighted case

The algorithm we use for the paper was first presented by Eades et al. (1993). This algorithm was chosen because it has a linear run time complexity, $O(m + n)$, and because of its relative implementation simplicity. The algorithm uses a greedy heuristic through which it builds the proposed ordering $s = s_{left}s_{right}$.[21] Vertices are initialized into several buckets: sinks, sources, and δ buckets, where for a vertex $u \in V$, $\delta(u) = d^{-}(u) - d^{+}(u)$.[22] At each iteration, the algorithm removes all sinks from the network and prepends them to a sequence s_{right}, removes all sources and appends them to a sequence s_{left}, and then removes the vertex with the lowest δ score (the most "source"-like vertex) and appends it to s_{left}.[23] Each removal requires updating the buckets to reflect the modified graph. The algorithm stops when the graph is empty. There will be $2n - 1$ buckets, which can be formalized as follows:[24]

$$\begin{aligned} V_{-n+1} &= V_{sources} = \{u \in V \mid d^{-}(u) = 0;\ d^{+}(u) > 0\} \\ V_{n-1} &= V_{sinks} = \{u \in V \mid d^{+}(u) = 0\} \\ V_d &= \{u \in V \mid d = \delta(u);\ d^{+}(u) > 0;\ d^{-}(u) > 0\} \end{aligned}$$

F.4 Example execution on unweighted network

Consider the following network:



Let's trace the execution of the algorithm described by Eades et al.

F.4.1 Initialization

Buckets:

[21] According to Black (2005), a greedy algorithm is, "An algorithm that always takes the best immediate, or local, solution while finding an answer. Greedy algorithms find the overall, or globally, optimal solution for some optimization problems, but may find less-than-optimal solutions for some instances of other problems."

[22] We have flipped the sign here compared to Eades et al. to be consistent with the diagrams elsewhere in our paper.

[23] Eades et al. take the vertex with the maximum δ score.

[24] Eades et al. assume that the graph G is simple (no bidirectional edges), and hence their original algorithm only requires $2n - 3$ buckets.

A			D	C	B			E
sources	-3	-2	-1	0	1	2	3	*sinks*

$Ordering: \ s = s_{left} = s_{right} = ()$

F.4.2 First iteration:

Remove sinks

Updated buckets:

A				C, D	B			
sources	-3	-2	-1	0	1	2	3	*sinks*

$Updated\ ordering: \ s_{left} = (),\ s_{right} = (E),\ s = s_{left}s_{right} = (E)$

Remove sources

Updated buckets:

				C, D, B				
sources	-3	-2	-1	0	1	2	3	*sinks*

$Updated\ ordering: \ s_{left} = (A),\ s_{right} = (E),\ s = s_{left}s_{right} = (A, E)$

Remove vertex with lowest delta score

Updated buckets:

B								D
sources	-3	-2	-1	0	1	2	3	*sinks*

$Updated\ ordering: \ s_{left} = (A, C),\ s_{right} = (E),\ s = s_{left}s_{right} = (A, C, E)$

F.4.3 Second iteration

Remove sinks

Updated buckets:

B								
sources	-3	-2	-1	0	1	2	3	*sinks*

$Updated\ ordering:\ s_{left} = (A, C),\ s_{right} = (D, E),\ s = s_{left}s_{right} = (A, C, D, E)$

Remove sources

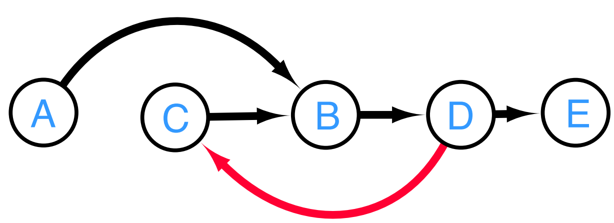
Updated buckets:

sources	-3	-2	-1	0	1	2	3	*sinks*

$Updated\ ordering:\ s_{left} = (A, C, B),\ s_{right} = (D, E),\ s = s_{left}s_{right} = (A, C, B, D, E)$

F.4.4 Final output

$Ordering:\ s = s_{left}s_{right} = (A, C, B, D, E),\ Violator\ edge\ set\text{: } \{(D, C)\}$



F.5 Weighted case

Simpson et al. (2016) have proposed a modification to adapt the Eades algorithm to solve the weighted problem. The required changes are:

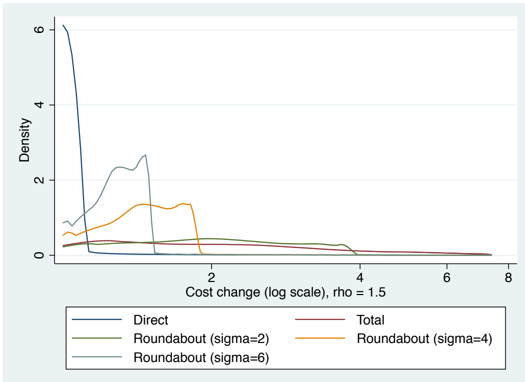
1. In the initialization step, all edge weights need to be normalized to be between 0 and 1.

2. $\delta(u)$ is redefined as $\delta(u) = \lfloor w^-(u) - w^+(u) \rfloor$.

The key motivation behind these steps is to reformat the network so that the unweighted version of the algorithm could be used in an identical fashion as before, specifically without increasing the number of buckets. Without the floor in step 2, for any given network the number of buckets could become large.

G Estimation results

Figure 8: Distributions of $\log \hat{c}$ from banning imports, different parameter values



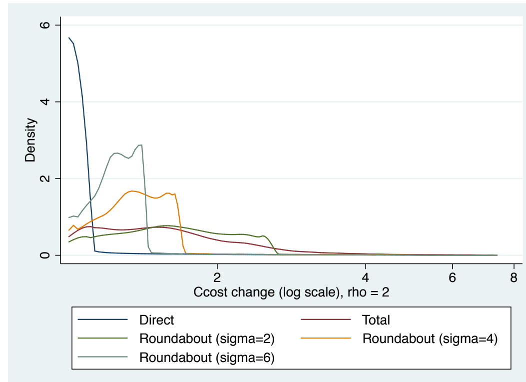


Table 14 shows the median value of cost change $\hat{c}_i$ under different parameter values.

Table 14: Median $\hat{c}_i$ under different values of σ and ρ

	Direct	Total	Roundabout		
			$\sigma = 2$	$\sigma = 4$	$\sigma = 6$
$\rho = 1.5$	1	2.89	2.37	1.53	1.33
$\rho = 2$	1	1.70	1.78	1.41	1.28

Table 15: 90th percentile $\hat{c}_i$ under different values of σ and ρ

	Direct	Total	Roundabout		
			$\sigma = 2$	$\sigma = 4$	$\sigma = 6$
$\rho = 1.5$	1.08	11.93	3.76	1.85	1.50
$\rho = 2$	1.04	3.45	2.52	1.67	1.42

Table 16: Change in price index $\hat{P}$ under different values of σ and ρ

ρ	σ	$\hat{P}\mid_{direct}$	$\hat{P}\mid_{total}$	$\hat{P}\mid_{roundabout}$
1.5	2	1.67	3.84	3.66
1.5	4	1.25	2.25	1.82
1.5	6	1.16	1.83	1.48
2	2	1.47	2.35	2.51
2	4	1.21	1.77	1.65
2	6	1.14	1.56	1.41

Table 17: Change in price index $\hat{P}$ under acyclic network

ρ	σ	$\hat{P}\mid_{total}$	$\hat{P}\mid_{total}$, acyclic
1.5	2	3.84	3.93
1.5	4	2.25	2.28
1.5	6	1.83	1.84
2	2	2.35	2.37
2	4	1.77	1.78
2	6	1.56	1.57

Notes: The fourth column shows the change in price index from banning imports when taking into account the acyclic network structure. We obtain the acyclic network from the algorithm explained in Appendix F, for the weighted case where we minimize the value of violating transactions.